CSCE 2022 Annual Conference
Whistler, British Columbia
May 25th – 28th, 2022

Parametric Study of a Cable Dome of Geiger-Type

Tulloch, Hannah[1], Ahmed, Elshaimaa[1], El Damatty, Ashraf[1,2]
[1] The University of Western Ontario, Canada
[2] damatty@uwo.ca

Abstract: Cable and membrane structures are typically geometrically flexible and are subject to larger deflections under loads when compared to geometrically rigid structures. As one of the well-known types of cable roofs, the preliminary design of cable domes requires assigning appropriate prestress to the cables and structs taking into account the strength, stability and serviceability requirements under different load conditions according to design codes. The ideal prestress is assigned initially based on the geometrical arrangements of cables and struts, then magnified according to the load carrying capacity of the dome. The arrangement of cables and struts can affect the ideal prestress and, eventually, the total weight and maximum displacement of the dome under service loads. In this regard, this study performs a parametric study to investigate the sensitivity of some geometrical parameters (radial divisions and number of sectors) to the maximum displacement and total weight of a cable dome of Geiger-type. A code was developed to determine the initial prestress of 12 domes (with configurations of 2 hoops and 10 sectors up to 4 hoops and 20 sectors) then further modeled using SAP2000 and subjected to service loads according to the NBCC code. The results of this study showed that the total weight and maximum displacement remain relatively constant when increasing the number of sectors. On the other hand, increasing the number of hoops leads to significantly less displacement and a heavier dome. Based on these results, it can be concluded that domes with a larger number of hoops perform better under external loading, but resultantly are heavier and therefore more expensive.

Keywords: parametric study, Geiger cable dome, ideal prestress

1 INTRODUCTION

Cable structures are incorporated in the design of long span buildings due to their lightweight and versatile nature and can be classified into three categories (Quagliaroli et al., 2015). The first is the pure tensile structure, in which one set of cables provides support while the other is for stabilization. The second is the tensegrity structure; a combination of both cables and struts that are self-equilibrated through prestressing and do not require the stiffness provided by supports. The third category of cable structures is the hybrid tensile structure, which is considered a tensegrity structure, but with perimeter support (Quagliaroli et al., 2015). Cable domes, as a hybrid tensile structure, attain their stability by assigning the appropriate level of prestress. Meanwhile, the initial geometry can affect greatly the prestress distribution and, consequently, the static behavior of cable domes under external loads. This phenomenon was touched on in some past studies, such as, Kawaguchi (1999) who studied the effects of changing the length of the outer-most post and found that increasing the length would decrease the vertical displacement of cable domes. Also, Quagliaroli et al. (2015), Yuan and Dong (2002), and Pollini (2021) investigated effects of the member's weight on the dome feasibility and determined the optimal weight for a feasible design. Castro and Levy (1992) found that increasing the strut height ultimately minimized the cost of the Georgia Olympic Dome, and that a two-hoop dome was more economical than a three-hoop one. Fu (2005) determined that a wedge-shaped cable network resulted in a lighter weight structure than a triangulated network as there was a smaller number of cable elements. Krishnan (2015) investigated the effects of the number polygon sides on torsional stiffness and found that domes with fewer sides were prone to torsional distortion. Based on the above and due to the growing demand in the market on this type of light-weight roofing, more research is required to cover further aspects in this domain.

The objective of the current research is to determine the optimal member arrangement for the design of long-span cable dome structures to the minimum weight and displacement. In this regard, this paper investigates the results of twelve cable dome models, each with different numbers of sectors and hoop cables and is subjected to various load combinations. The paper is organized as follows. Section 2 includes the design of positive curvature cables domes by, first, determining prestresses using the Singular Value Decomposition method, then, calculating the required cross-sectional areas under different load conditions according to the NBCC code. Twelve study cases are presented. Section 3 presents and discusses the results of the numerical model. Finally, Section 4 concludes the results.

2 DESIGN PROCEDURE OF A CABLE DOME OF GEIGER TYPE

2.1 Calculating Prestress of Cable Domes Using Singular Value Decomposition

The fundamental contributions of matrix analysis of pin-jointed tensegrities using the Singular-Value Decomposition (SVD) method are due to the work done by Pellegrino (1993). This method is based on calculating the connectivity matrix of the dome as illustrated in Tran et al. (2012), then calculating the projected lengths of all members in the x-, y-, and z- directions as follows,

[1] $l^x = Cx + C_f x_f$
[2] $l^y = Cy + C_f y$
[3] $l^z = Cz + C_f z_f$

Where (x, y, z) and (x_f, y_f, z_f) are the nodal coordinates for the free and fixed nodes in x, y, z directions, respectively. C and C_f describe the connectivity of the members to the free and fixed nodes, respectively. The members lengths are then calculated using,

[4] $l = \sqrt{lx^2 + ly^2 + lz^2}$

The equilibrium matrix A can be formed using the project lengths as defined by,

[5] $A = \begin{pmatrix} C^T diag(l^x) \\ C^T diag(l^y) \\ C^T diag(l^z) \end{pmatrix} L^{-1}$

Where L is diagonal *l*. By applying the SVD technique on the equilibrium matrix, the vector of prestress of all members can be retrieved. The unilateral conditions of all members, i.e., struts are under compression and cables under tension, should be checked, otherwise, the geometry should be changed. A code was developed using MATLAB that reads the connectivity matrix of the dome, constructs the equilibrium matrix, and performs SVD technique to determine prestresses for all domes.

The prestress of one of the elements in the inner ring of the dome is scaled to 1, then the prestresses of all other elements are proportionally scaled up by the same ratio. Those values are then magnified to 10^8 *N* which achieves the minimum displacement of the dome under external loads.

2.2 Load Cases

Krishnan (2015) examined the various loads and load combinations that cable domes are designed to withstand. These loads include prestressing forces, dead and live loads, snow loads, and wind loads, which are included in the design of twelve cable domes. Prestressing forces are necessary to ensure that all cables remain in tension and that the deflections are within the designated limits. Prestressing loads depend on the geometry, member size and deflection limits of the structure. The dead loads, live loads, snow loads, and wind loads are applied at the upper joints based on the tributary area each node supports.

Loads considered:

1) live load L=1.0 kN/m^2
2) snow load S=0.85 kN/m^2
3) wind load W=0.9 kN/m^2 in suction
4) prestress load St

Load combinations:

The dome models are subject to load combinations from Table 4.1.3.2.-A of the National Building Code of Canada as presented in Table 1, with the addition of the prestress load to each combination. The allowable stress was taken as 30% of critical tensile strength for cables, and 60% of critical compressive strength for the struts (Wang et al. 2010). The critical strength of cables and struts are $\sigma_c = 1.67 \times 10^6\ kN/m^2$ and $\sigma_s = 3.45 \times 10^5\ kN/m^2$, respectively and the Young's Modulus for cables and struts are $E_c = 1.9 \times 10^8\ kN/m^2$ and $E_s = 2.06 \times 10^8\ kN/m^2$, respectively.

Table 1: Load Combinations from NBCC

Case	Principal Loads	Companion Loads
1	1.4**D**+**St**	-
2	(1.25**D** or 0.9**D**)+1.5**L**+**St**	1.0**S** or 0.4**W**
3	(1.25**D** or 0.9**D**)+1.5**S**+**St**	1.0**L** or 0.4**W**
4	(1.25**D** or 0.9**D**)+1.4**W**+**St**	0.5**L** or 0.5**S**

2.3 Study Cases

Twelve cable domes are considered in the current study. The domes range from two hoops and ten sectors (2H10S) to four hoops and twenty sectors (4H20S), as shown in Figure 1 to 12. The prestresses for the domes are first determined using a code developed in MATLAB based on SVD method as outlined in Section 2.1. All domes were then designed using the commercial software SAP2000 according to the envelope of all load combinations outlined in section 2.2.

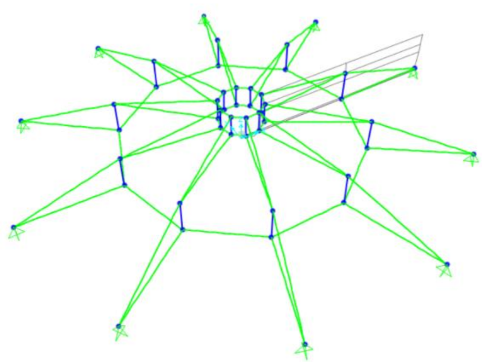

Figure 1: 2 Hoops 10 Sectors

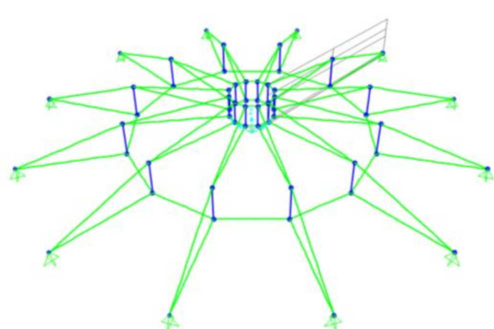

Figure 2: 2 Hoops 12 Sectors

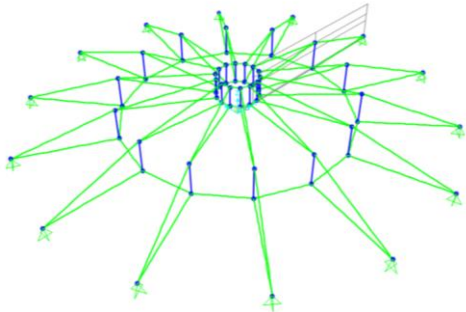

Figure 3: 2 Hoops 14 Sectors

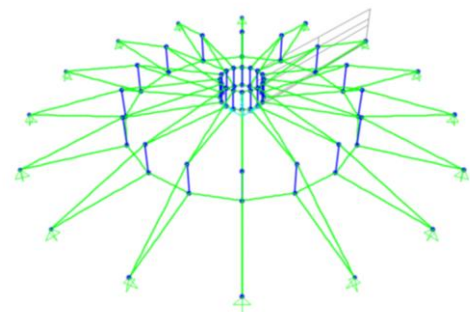

Figure 4: 2 Hoops 16 Sectors

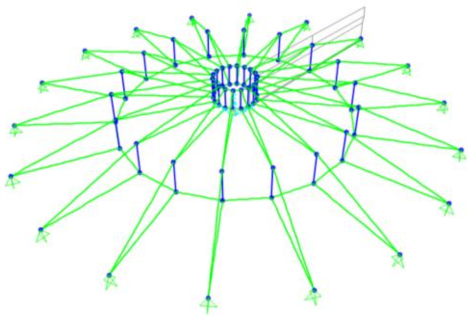

Figure 5: 2 Hoops 18 Sectors

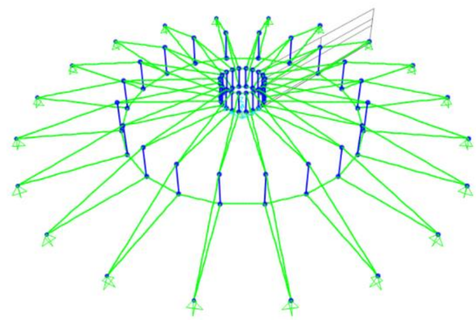

Figure 6: 2 Hoops 20 Sectors

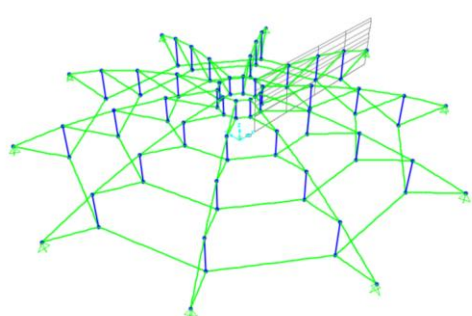

Figure 7: 4 Hoops 10 Sectors

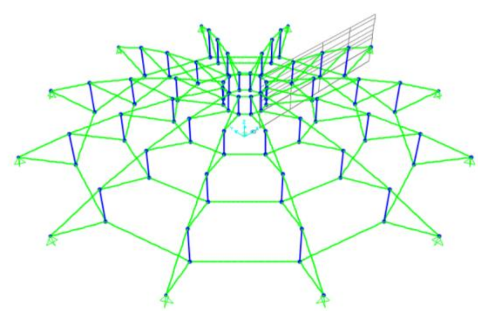

Figure 8: 4 Hoops 12 Sectors

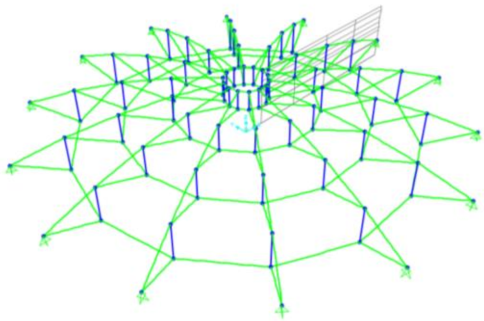

Figure 9: 4 Hoops 14 Sectors

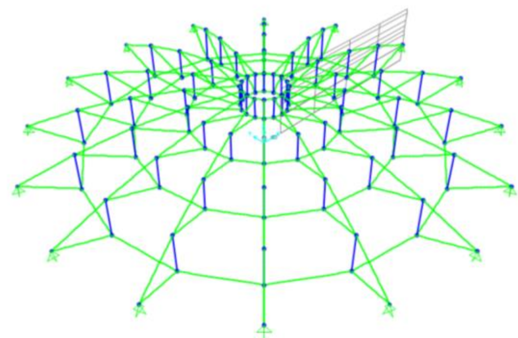

Figure 10: 4 Hoops 16 Sectors

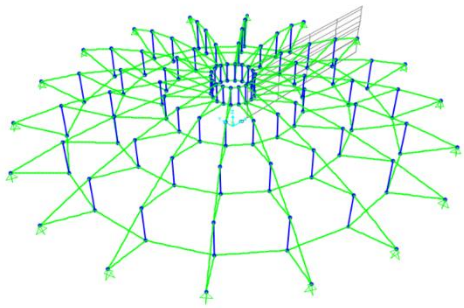

Figure 11: 4 Hoops 18 Sectors

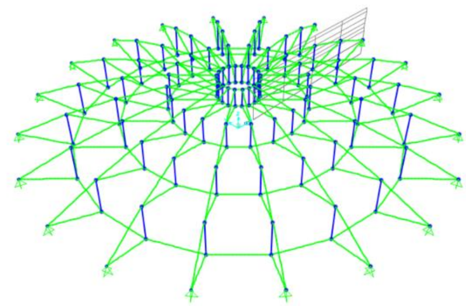

Figure 12: 4 Hoops 20 Sectors

3 RESULTS AND DISCUSSION

3.1 Prestress Results

As the first step in designing cable domes, the feasible prestress of all members should be determined before external loads can be applied and then magnified according to the load carrying capacity of the domes. Table 2 and 3 compare the prestresses of all groups of members in models with 2 hoops and 4 hoops, respectively. This comparison showed that the prestress distribution in the hoop cables doesn't change significantly when changing the number of sectors. However, the prestresses in both diagonal, ridge cables decrease with the increase in the number of sectors. For example, the prestress for the HS0 cable in two hoops and ten sectors (2H10S) is 1.37778 and for two hoops and twenty sectors (2H20S), the prestress is 1.375. Figure 13 and 14 illustrate the geometry and group members of cable domes with two and four hoops, respectively. Contrary to this, the number of hoops does affect the prestress distribution. For example, HS0 cable in two hoops and fourteen sectors (2H14S) dome has a prestress of 1.37805 while the same cable in four hoops and fourteen sectors (4H14S) has a prestress of 0.25993.

3.2 Total Weight Results

By comparing the total weight from all models, it can be seen that the weight is almost the same for all models with the same number of hoop cables as shown in Figure 15 and tabulated in Table 14. This equivalency is due to the fact that increasing the number of sectors decreases the tributary area each node supports, and this decreases the cross-sectional areas of the elements in each sector. As a result, the total weight of the dome remains almost the same when increasing the number of sectors keeping the number of hoops unchanged. In other words, the increased number of sectors balances the decreased values of cross-sectional areas. For example, the total weight of the dome with four hoops and ten sectors (4H10S) is 11800.73 KN, meanwhile the dome with four hoops and twenty sectors (4H20S) weighs 12358.71 KN. Although they differ significantly in the number of members, they have very similar total weights.

3.3 Total Displacement Results

By comparing the total displacement of all domes, the number of hoop cables has the greatest effect on the total displacement as shown in Table 4 and Figure 16. As the number of hoops increases, the total weight increases, while the maximum displacement decreases. This can be interpreted as that the dome becomes stiffer when increasing the number of hoops, which confines the domes. For example, the max displacement for the dome with two hoops and sectors sectors (2H14S) is 0.028921 m, while the dome with four hoops and fourteen sectors (4H14S) undergoes a max displacement of 0.005015 m. Although both domes have the same number of sectors, the dome with four hoops has a significantly smaller max displacement. On the other hand, the number of sectors does not affect greatly the max displacement. As shown in Table 4, four hoops and ten sectors (4H10S) dome has a maximum displacement of 0.005232m, while the dome with four hoops and twenty sectors (4H20S) has a maximum displacement of 0.005624m. Although the number of sectors has changed, the maximum displacement remained nearly constant.

3.4 Effect of Outermost Struts' Length

When comparing the length of outermost struts of both domes, shown in Figure 13 and 14, it is obvious that the outermost struts in the domes with two hoops are relatively short compared to vertical elevation of supports. Therefore, the diagonal cables are carrying excessive stresses and have higher prestresses compared to the ridge cables. To illustrate further, the diagonal cable XS2 in the dome with two hoops and ten sectors has a prestress of 18.4222, whereas the ridge cable JS2 has a prestress of only 1.54444, as shown in Table 2. As a result, cable XS2 is carrying much of the load and, accordingly, must have a larger cross section than if the outer post dropped lower. On the same line, increasing the length of the outer-most struts can redistribute the forces among the diagonal cables, ridge cables and the outermost struts. This reflects the sensitivity of this type of structures to the geometry configuration, in general, and the length of struts, in particular. The effects of the lengths of the outermost struts were investigated by Kawaguchi et al. (1999), where they illustrated that increasing the lengths of the outermost struts can decrease the vertical displacement by 25-35%.

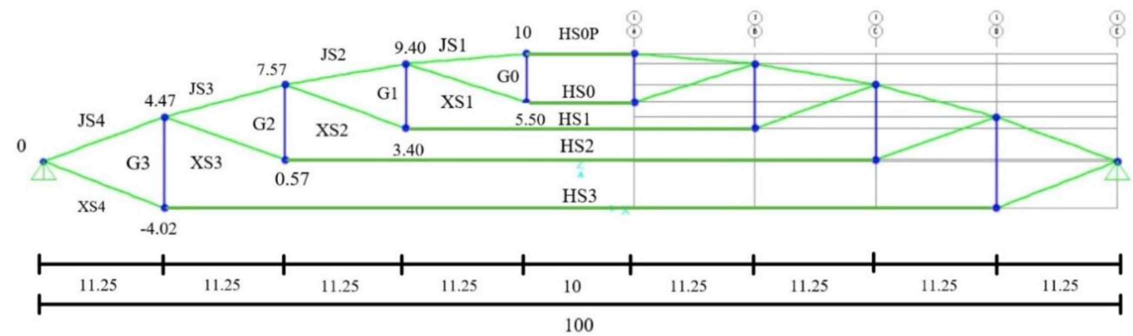

Figure 13: Model Geometry for 2 Hoops

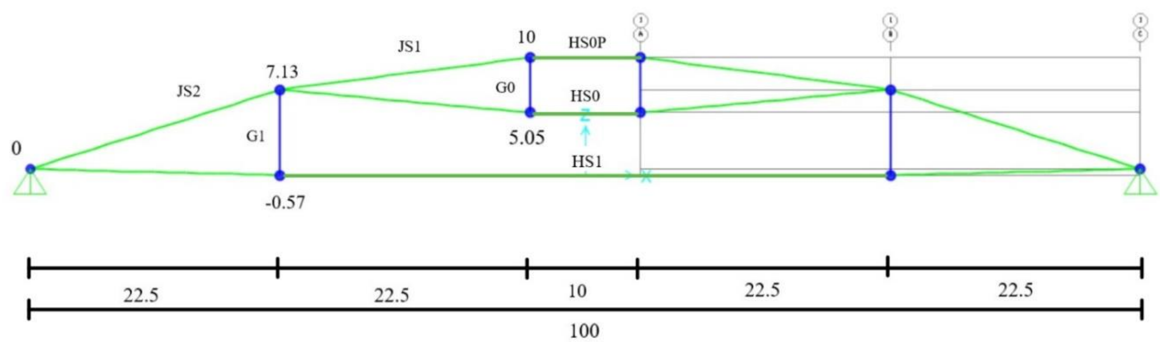

Figure 14: Model Geometry for 4 Hoops

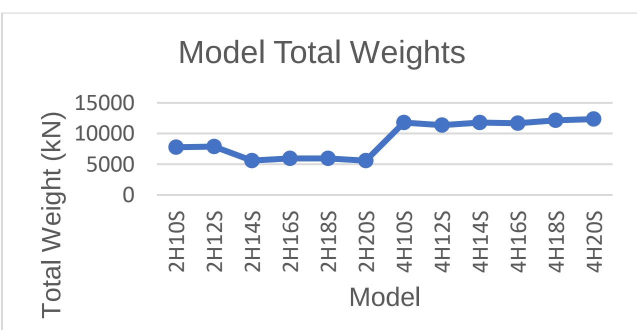

Figure 15: Model Total Weights

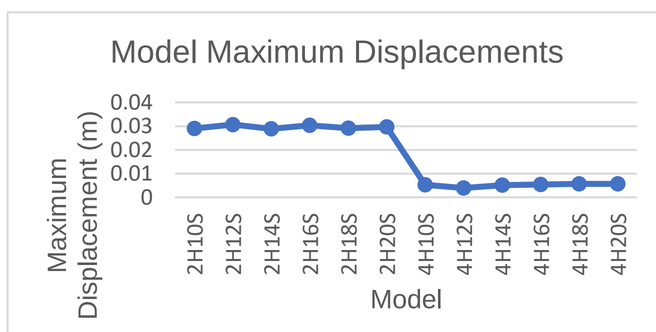

Figure 16: Model Maximum Displacement

Table 2: Prestress Modes for 2 Hoops

Member	10S	12S	14S	16S	18S	20S
HS0P	1	1	1	1	1	1
HS0	1.37778	1.38372	1.37805	1.38462	1.37333	1.375
HS1	29.8111	29.7326	29.7073	29.7949	29.6267	29.5833
JS1	0.62222	0.52326	0.45122	0.39744	0.34667	0.31944
JS2	1.54444	1.2907	1.10976	0.97436	0.86667	0.77778
XS1	0.85556	0.72093	0.60976	0.53846	0.48	0.43056
XS2	18.4222	15.3953	13.2195	11.6282	10.2933	9.26389
G0	-0.07778	-0.06977	-0.06098	-0.05128	-0.04	-0.04167
G1	-0.46667	-0.39535	-0.32927	-0.29487	-0.26667	-0.23611

Table 3: Prestress Modes for 4 Hoops

Member	10S	12S	14S	16S	18S	20S
HS0P	1	1	1	1	1	1
HS0	0.26003	0.26094	0.25993	0.25976	0.26003	0.26079
HS1	0.60193	0.6013	0.60265	0.60272	0.60209	0.60252
HS2	1.39647	1.39546	1.39735	1.39728	1.39616	1.39748
HS3	3.08026	3.07942	3.08278	3.08149	3.07853	3.08094
JS1	0.61958	0.51864	0.44702	0.39219	0.34904	0.31475
JS2	0.78973	0.66126	0.56954	0.49915	0.44328	0.39928
JS3	1.19101	0.99676	0.85762	0.75212	0.66841	0.60252
JS4	2.14607	1.79579	1.54636	1.35484	1.20593	1.08633
XS1	0.16854	0.141	0.12252	0.10696	0.09424	0.08633
XS2	0.39647	0.33063	0.28477	0.24958	0.22164	0.19964
XS3	0.91653	0.76661	0.6606	0.57895	0.51483	0.46403
XS4	2.04173	1.70989	1.47185	1.29032	1.1466	1.03417
G0	-0.05136	-0.04214	-0.03642	-0.03226	-0.02792	-0.02518
G1	-0.13483	-0.11183	-0.09603	-0.08489	-0.07504	-0.06835
G2	-0.30658	-0.2577	-0.22185	-0.19355	-0.17277	-0.15468
G3	-0.73997	-0.61912	-0.53311	-0.46689	-0.41536	-0.3741

Table 4: Model Total Weights and Maximum

Model	Weight (KN)	Max. Disp. (m)	Model	Weight (KN)	Max. Disp. (m)
2H10S	7775.255	0.028975	4H10S	11800.73	0.005232
2H12S	7876.44	0.030687	4H12S	11358.88	0.003917
2H14S	5609.393	0.028921	4H14S	11772.08	0.005104
2H16S	5930.314	0.030327	4H16S	11659.94	0.005427
2H18S	5977.006	0.029123	4H18S	12130.15	0.005634
2H20S	5616.371	0.029741	4H20S	12358.71	0.005624

4 CONCLUSION

This paper investigated the effects of the number of hoops and sectors on the total weight and maximum displacement of long span cable dome structures. Twelve domes were designed under different load combinations and compared in terms of the max displacement, total weight and prestress distribution. By analyzing the results, it can be concluded that changing the number of sectors has the least effect on the total weight and the maximum displacement of the dome, rather, increasing the number of hoops stiffens the structure by decreasing the total displacement, despite the increase in weight. Moreover, the behavior of the dome is highly affected by the length of struts, especially when the outermost struts are relatively short compared to the vertical elevation of supports, leading to excessive stresses in the diagonal cables compared to the ridge cables. This reflects the sensitivity of this type of structures to the geometry configuration, in general, and the length of struts, in particular.

Acknowledgments

I would like to extend my thanks to Dr. Ashraf El Damatty and Elshaimaa Ahmed for their support and guidance during the duration of this research, and without whom this paper would not be possible.

References

Castro, G. and Levy, M.P., 1992, June. Analysis of the Georgia Dome cable roof. In *Computing in Civil Engineering and Geographic Information Systems Symposium* (pp. 566-573). ASCE.

Fu, F., 2005. Structural behavior and design methods of tensegrity domes. *Journal of Constructional Steel Research*, *61*(1), pp.23-35.

Kawaguchi, M., Tatemichi, I. and Chen, P.S., 1999. Optimum shapes of a cable dome structure. *Engineering Structures*, *21*(8), pp.719-725.

Krishnan, S., 2015. *Prestressed cable domes: structural behavior and design*. University of Illinois at Urbana-Champaign.

Pellegrino, S., 1993. Structural computations with the singular value decomposition of the equilibrium matrix. *International Journal of Solids and Structures*, *30*(21), pp.3025-3035.

Pollini, N., 2021. Gradient-based prestress and size optimization for the design of cable domes. *International Journal of Solids and Structures*, *222*, p.111028.

Quagliaroli, M., Malerba, P.G., Albertin, A. and Pollini, N., 2015. The role of prestress and its optimization in cable domes design. *Computers & Structures*, *161*, pp.17-30.

Tran, H.C., Park, H.S. and Lee, J., 2012. A unique feasible mode of prestress design for cable domes. *Finite Elements in Analysis and Design*, *59*, pp.44-54.

Wang, Z., Yuan, X. and Dong, S., 2010. Simple approach for force finding analysis of circular Geiger domes with consideration of self-weight. *Journal of Constructional Steel Research*, *66*(2), pp.317-322.