University of Massachusetts Amherst

Design of a Reinforced Concrete Beam in a Continuous Frame

CEE 433 - Spring 2021

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Introduction:

We were tasked with designing a second floor beam of the frame on line C for a five story building. We were to design the beam's dimensions, number and arrangement of longitudinal bars, bar sizes, cutoff locations, and stirrup locations.

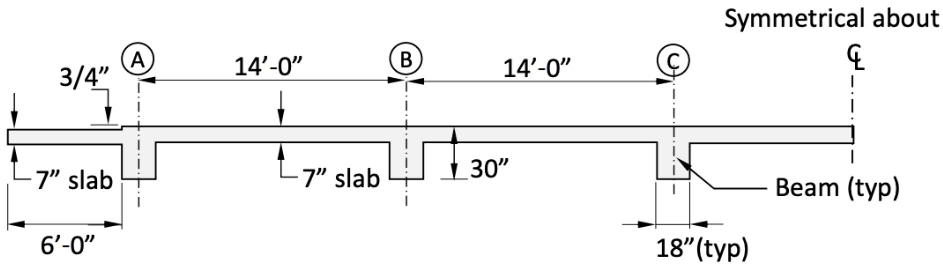
Procedure:

Loads were calculated dead, wind, and live for the second story frame. The building frame was then made into **SAP 2000** to be analyzed. Shear and moment diagrams were made with load combinations. The reinforcement was then designed based on the maximum moment found in the moment diagrams. The stirrups were designed based off of the shear diagram that was made in sap. The longitudinal cutoffs were made for the reinforcement and the area of steel.It was assumed for longitudinal bars;

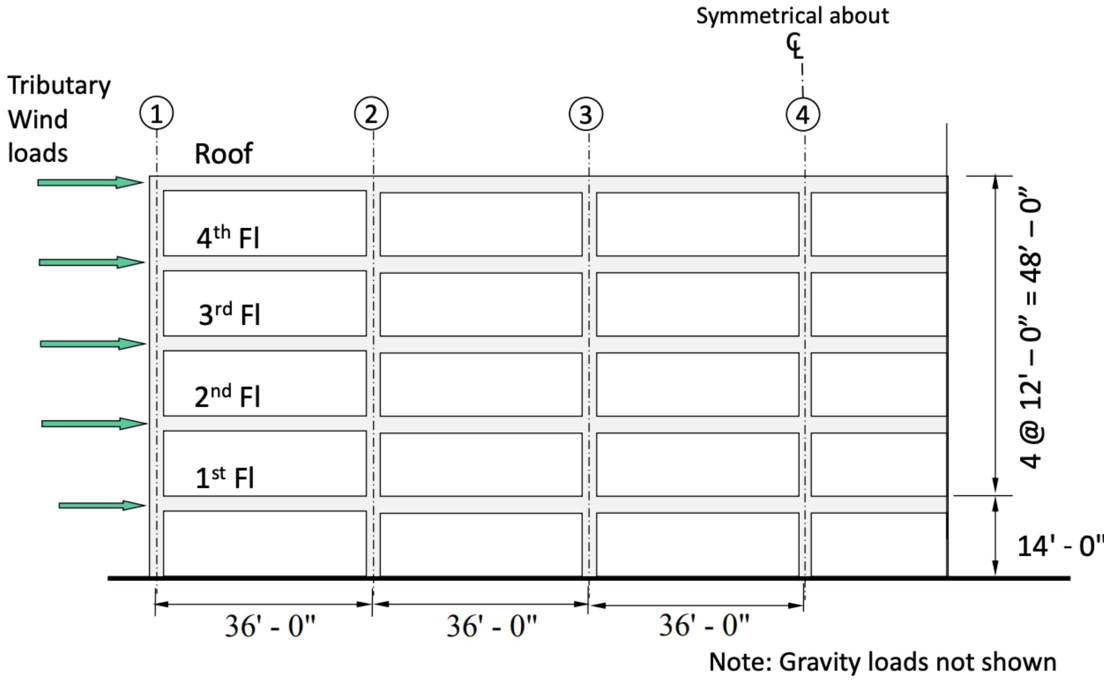
$$l_d = \frac{f_y}{\lambda\sqrt{f'_c}} \cdot \frac{\psi_t \cdot \psi_e}{20} d_b \qquad \text{with} \qquad \psi_e \text{ and } \lambda = 1.0$$

The capacity of the bars were checked to see if the selected bars chosen did not exceed the maximum capacity of the bars used in design.Lastly the beam is drawn to show the designs that we have made.

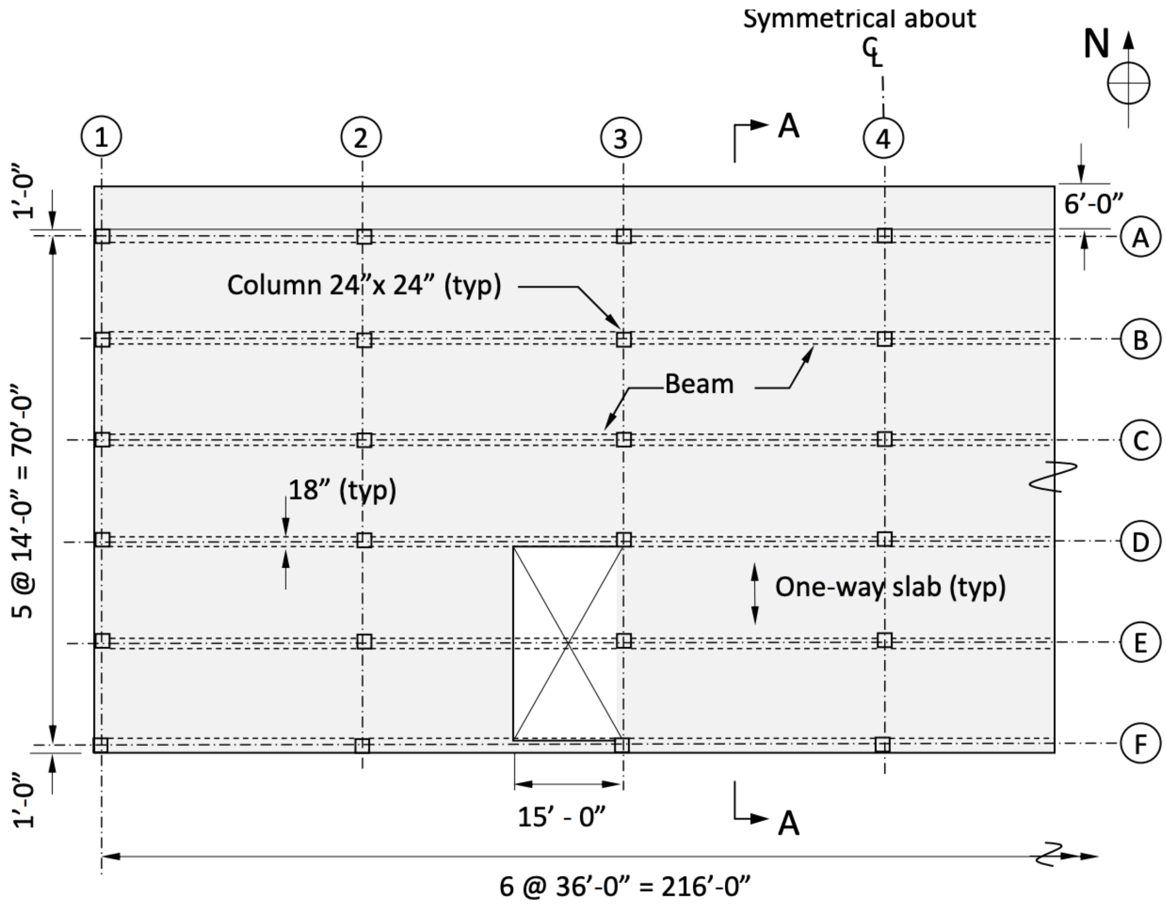
The figures below show the structural system of a reinforced concrete office building. Each frame in the building along lines A through F support the tributary gravity loads generated on the one-way slab of the building in addition to the wind load acting in the east- west direction on the side of the building



Section A-A



Building Elevation



Typical Floor Plan

Calculations:

The reinforcement at different sections along the second floor beam for the critical moments computed from analysis (positive and negative)

Load calculations:

Self weight : (SAP2000, will calculate the self weight)

Super imposed : $\frac{15\,lb}{ft^2} \times$ Tributary width

$$\frac{15\,lb}{ft^2} \times 14\,ft \times \frac{1\,kip}{1000\,lb} = \boxed{0.21\ kip/ft}$$

Typical Floor Live load: $\frac{65\,lb}{ft^2} \times 14\,ft \times \frac{1\,kip}{1000\,lb} = \boxed{0.91\ kip/ft}$

Roof Live load : $\frac{20\,lb}{ft^2} \times 14\,ft \times \frac{1\,kip}{1000\,lb} = \boxed{0.28\ kip/ft}$

Wind Load:

Wind load For Roof: $\frac{32\,lb}{ft^2} \times$ tributory Area

$$\frac{32\,lb}{ft^2} \times 14\,ft \times 6\,ft \times \frac{1\,kip}{1000\,lb} = \boxed{2.69\ kip}$$

Wind load for 1^{st} floor: $\frac{32\,lb}{ft^2} \times 14\,ft\,(6+7)\,ft \times \frac{1\,kip}{1000\,lb} = \boxed{5.82\ kip}$

Wind load For floor (2^{nd} – 4^{th}) : $\frac{32\,lb}{ft^2} \times 14\,ft \times 12\,ft \times \frac{1\,kip}{1000\,lb} = \boxed{5.38\ kip}$

$f'c = 5000psi$ $W_L = 0.91 Kip/ft$ $b = 18''$
$fy = 60000psi$ $W_D = 0.77 Kip/ft$ $h = 30''$
$W_R = 0.28 Kip/ft$
$W_W = 5.38 Kip/ft$ (2nd floor)

Self weight $= 18'' \times 30' \times 150 lb/ft^3 \times \frac{1}{144} = 562.5 lb/ft$

$W_D = 0.562 K/ft + 0.21 K/ft = 1.542$

$W_u = 1.2D + 1.0W + 1.0L + 0.5Lr = 7.35 Kip/ft$

$Mu = \frac{Wu l^2}{8} = \frac{7.354 K/ft \times 36^2}{8} = 1191.35 kip\text{-}ft$

$Mu \leq \phi Mn$

$Mn,req = \frac{1191.35 \times 12 in/ft}{0.9} = 15884.64 kip\cdot in$

$\rho max = \frac{(0.85 f'c)}{fy} \frac{3}{8} \beta_1 = 0.0212$

$0.70 \rho max = 0.0149$

$Rn = \rho fy(1 - 0.5 \rho m) = 0.0149 \times 60 ksi (1 - 0.5(0.0149)(14.12))$

$m = \frac{fy}{0.85f'c} = \frac{60}{0.85(5)} = 14.12$

$Rn = 0.799 ksi$

$bd^2 = \frac{Mu}{\phi Rn} = \frac{266.6 \times 12 in/ft}{0.9 \times 0.799} = 4448.89 in^2$

use $Mu = -266.6$ Kip-ft

From SAP 2000

$M_R = -266.6$ Kip-ft
M_C ~~132~~ 133.42 Kip-ft
M_L -239.27 Kip-ft

b	d
10	21.09
12	19.25
14	17.83
16	16.67

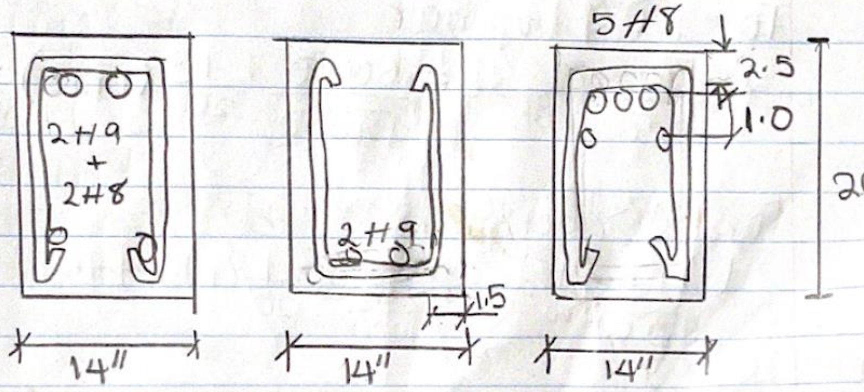
use 14" × 20"
$d = 20'' - 1.5 + 3/8'' - 0.5$
$d = 17.5''$
Adjusted Moment F = 1.01

Use $R = \dfrac{M \times 12 \text{in/ft}}{0.9 \times 14'' \times 17.5^2}$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mRn'}{fy}}\right)$$

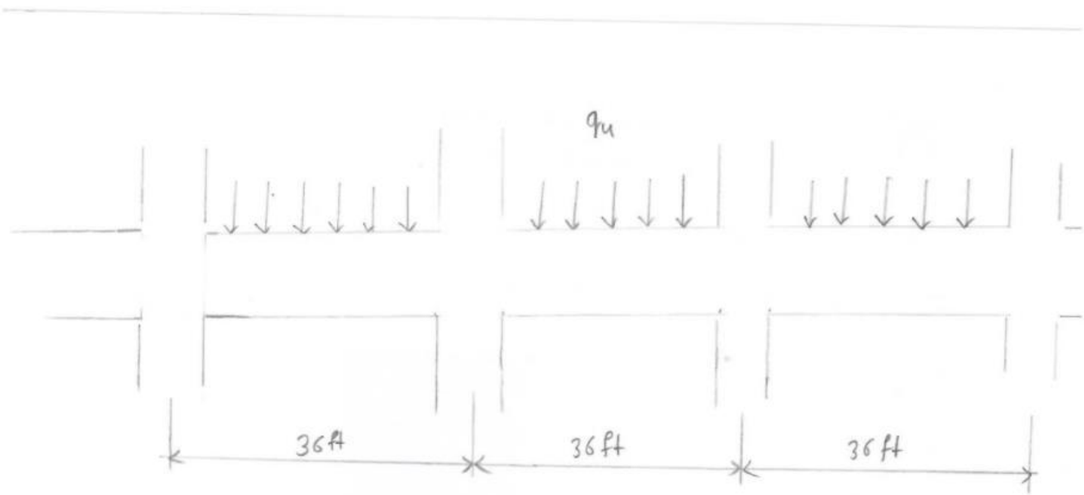
use $As = \rho \times b \times d$

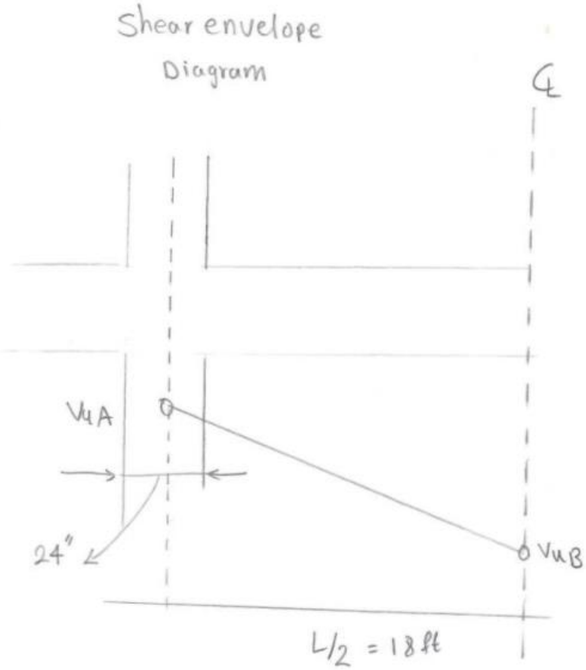
M	Adjusted moments	R	ρ	A (in²)	
L	241.60	0.75	0.014	3.43	2#9+2#8
C	134.75	0.42	0.0074	1.8	2#9 (1)
R	269.27	0.84	0.016	3.85	5#8 = 3.95 in²



left Centre Right

<u>The shear reinforcement (stirrups) needed along the span of the beams (second floors)</u>



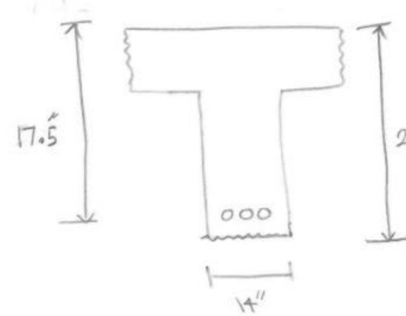


Property Materials

$f'_c = 5000 psi$

$f_y = 60000 psi$

$q_u = 1.2D + 1.0W + 1.0L + 0.5L_r$



$q_u = 1.2D + 1.6L$

Dead load for revised beam (self weight)

$$\frac{(20'')(14'')}{144\ in/ft^2}\left(150\ \frac{lb}{ft^3}\right)\cdot\left(\frac{1\ kip}{1000\ lb}\right) = 0.29\ kip/ft$$

Superimpose Load : 0.21 kip/ft

total dead load = selfweight + superimposed

$= 0.29 + 0.21$

$= 0.502\ kip/ft$

$$q_u = 1.2\left(0.502\ \frac{kip}{ft}\right) + 1.6\ (0.91) = 2.0584$$

$$V_{uA} = \frac{1}{2} q_u l = \frac{1}{2}(2.0584)(36) = 37\ kip$$

$$V_{uB} = \frac{1}{8} q_{u_L} l = \frac{1}{8}(1.6 \times 0.91)(36) = 6.56\ kip$$

$$\phi V_n \geq V_u$$

$$\phi V_n = \phi\ (V_c + V_s)$$

if $A_{v,min}$ is provided :

According to ACI 318-19 : assuming Normal weight concrete $\lambda = 1$

$$\begin{cases} V_c = 2\lambda\sqrt{f'_c}\, b_w d & \Rightarrow\ 2(1)\sqrt{5000}\ \dfrac{(14'' \times 17.5'')}{1000} \\ V_c = 8\lambda(\rho_w)^{1/3}\sqrt{f'_c}\, b_w d & \Rightarrow 34.65\,\text{kip} \end{cases}$$

$$V_c = 34.65\ \text{kip}$$

$$\phi V_c = 0.75\,(34.65\ \text{kip})$$

$$\phi V_c = 25.98\ \text{kip}$$

$$\phi V_s = V_u - \phi V_c$$

$$\phi V_s = V_u - 25.98\,\text{kip}$$

$$\text{Slope} = \frac{V_uA - V_uB}{L/2} = \frac{37 - 6.56}{18\ \text{ft}} = 1.69\ \text{kip/ft}$$

$$V_{u@face} = 37^k - 1.69\frac{k}{ft}\left(\frac{12''}{12'}\right) = 35.31\,k$$

Let's check if it satisfy the requirement:

$$V_u \leqslant \phi\left(V_c + 8\sqrt{f'_c}\ b_w d\right)$$

$$0.75\left(34.65^k + 8\sqrt{5000}\ \left(\frac{14'' \times 17.5''}{1000}\right)\right)$$

$$= 130\ \text{kip}$$

$$35.31^k < 130\,k \quad \checkmark$$

Table 9.7.6.2.2 - Maximum spacing of legs of shear reinforcement
ACI 318-14

if $V_s \leqslant 4\sqrt{f'_c}\, b_w d \Rightarrow s_{max} = d/2$; otherwise $d/4$

$V_s = 12.44^k$ yes ✓ $\rightarrow s_{max} = d/2$

$$4\sqrt{5000}\,\frac{(14'' \times 17.5'')}{1000} = 69.3$$

Minimum Transverse reinforcement requirements

if $V_u \geqslant \Phi\lambda\sqrt{f'_c}\, b_w d$

$$\Phi\lambda\sqrt{f'_c}\, b_w d = 0.75\,(1.0)\sqrt{5000}\,\frac{(14'' \times 17.5)}{1000}$$

$$\Phi\lambda\sqrt{f'_c}\, b_w d = 12.99$$

$V_u = 37.0\,kip$

Then, $V_u \geqslant \phi\lambda\sqrt{f'_c}\, b_w d$ $A_{v,min}$ required.

$$\frac{A_{v,min}}{s} = 50\,\frac{b_w}{f_y} = \frac{50\,(14'')}{60{,}000} = 0.010\ in^2/in$$

or

$$\frac{A_{v,min}}{s} = 0.75\,\frac{\sqrt{f'_c}}{f_y}\,b_w = 0.75\,\frac{\sqrt{5000}}{60{,}000}\,14'' = 0.012\ in^2/in$$

Stirrup Contribution (V_s)

$$\phi V_{s,req} = V_u - \phi V_c$$

X (in)	Vu (kip)	$\phi V_{s,req}$ (kip)	$V_{s,req}$ (kip)	S, calc (in)	S max (in)
12	35.31	9.33	12.44	18.57 in	d/2
36	31.93	5.95	7.93	29 in	d/2
72	26.86	0.88	1.173	196	d/2
⋮	⋮	⋮	⋮	⋮	⋮
216					

$$S = \frac{A_v f_y d}{V_s}$$ using No.3 stirrups (0.11 in²)

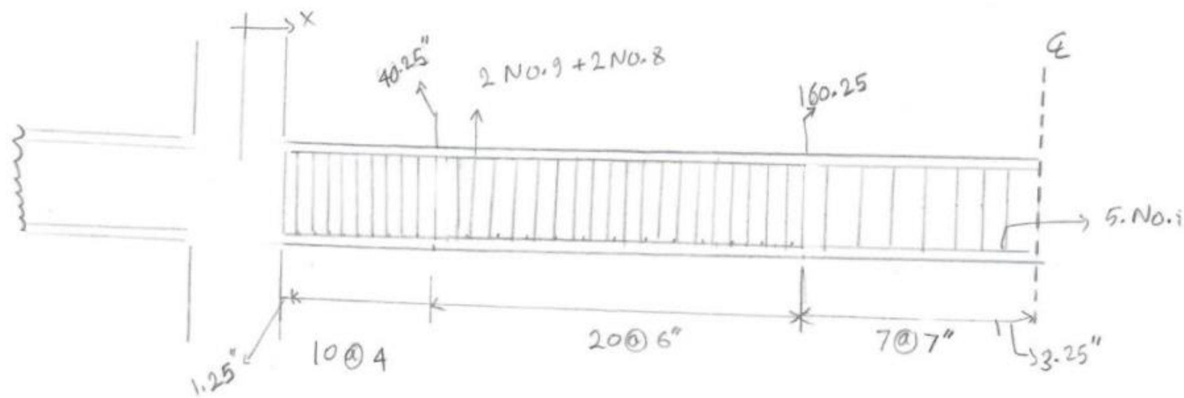
$$S = \frac{2(0.11\,in^2)(60ksi)(17.5)}{12.44}$$

$$S = \frac{A_v f_y d}{V_s}$$

No.3 Stirrups Area ($0.11 in^2$)

$$S = \frac{2\,(0.11 in^2)(60 ksi)(17.5)}{V_s} = \frac{231}{V_s}$$

$$S = \frac{(231)\,kip.in}{V_s}$$



Development of longitudinal bars for No.8 and No. 9 Bars

Capacity of Beam with Selected NO 9 Bars:

$A_s = 2\#9$

$$a = \frac{A_s \cdot f_y}{0.85 f'_c b_w} = \frac{2 \times 60}{0.85 \times 4 \times 14} = 2.52\,in$$

$$M_n = A_s \cdot f_y \cdot \left(d - \frac{a}{2}\right) = 2\,in^2 \times 60\,ksi \left(20 - \frac{2.52}{2}\right)\left(\frac{1}{12}\right)$$

$$M_n = 187.5\ kip\text{-}ft$$

$$\phi M_n = 0.9 \times (187.5)\,kip\text{-}ft = 168.75\,kip\text{-}ft$$

Development length of NO. 8 and NO. 9 bars

Bars are spaced at more than the $1.0 d_b$ minimum required. Assume minimum stirrups are present throughout development length

$$l_d = \frac{f_y}{\lambda \sqrt{f'_c}} \cdot \frac{\psi_t \cdot \psi_e}{20} d_b \quad \text{with } \psi_e \text{ and } \lambda = 1.0$$

For NO. 9 bottom bar:

$$l_d = \frac{f_y}{\sqrt{f'_c}} \cdot \frac{1}{20} d_b = \frac{60000}{\sqrt{4000}} \cdot \frac{1}{20} (1.128)\,in = 53.4\,in \simeq \boxed{54\,in}$$

For NO. 9 top bar

$$l_d = \frac{f_y}{\sqrt{f'_c}} \cdot \frac{1}{20} d_b = \frac{60000}{\sqrt{4000}} \cdot \frac{1.3}{20} (1.128) = 69.56\,in \simeq \boxed{70\,in}$$

For NO. 8 bottom bars

$$l_d = \frac{f_y}{\sqrt{f'_c}} \cdot \frac{1}{20} d_b = \frac{60000}{\sqrt{4000}} \cdot \frac{1}{20} (1) = 47.4\,in \simeq \boxed{48\,in}$$

For NO. 8 top bars

$$l_d = \frac{f_y}{\sqrt{f'_c}} \cdot \frac{1}{20} d_b = \frac{60000}{\sqrt{4000}} \cdot \frac{1.3}{20} (1) = 61.6\,in \simeq \boxed{62\,in}$$

(1) Check the feasibility for 2NO9 + 2NO8

$C = 0.85 f'c\, ba = 0.85(5)(14)a = 59.5a$

$T = A_s f_y = 3.58 \times 60 = 214.8$ k-ft

$a = \frac{T}{C} = 3.61$ in

$\phi M_n = 0.9(214.8)(17.5 - \frac{3.61}{2})\frac{1}{12} = 252.85$ kip-ft

$M_n = \frac{252.85}{0.9} = 280.94$ kip-ft

$\phi M_n = 252.85$ kip-ft $> M_u = 241.60$ kip-ft

For 2 NO 9

$C = 0.85 f'c\, ba = 0.85(5)(14)a = 59.5a$

$T = A_s f_y = 120$ kip-ft

$a = \frac{T}{C} = 2.02$

$\phi M_n = 148.41$ kip-ft

$\phi M_n = 0.9(120)(17.5 - \frac{1}{2})\frac{1}{12} = 148.41$ kip-ft

$M_n = 164.91$ kip-ft

$\phi M_n = 148.41$ kip-ft $> M_u = 134.75$ kip-ft

For 5 #8 $= 3.85 in^2$

$C = 0.85 f'c\, b\, a = 59.5a$

$T = A_s f_y = 3.85 \times 60 = 231$ k-ft

$a = \frac{T}{C} = 3.88$ in

$\phi M_n = 269.67$ kip-ft > 269.27 kip-ft (M_u)

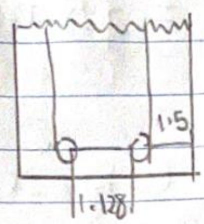
Check category

Spase = 2(1.128) + 4(1.128) + 2(1.5) + 2(1)

#9 Bars   Bar spacing   cover   #8

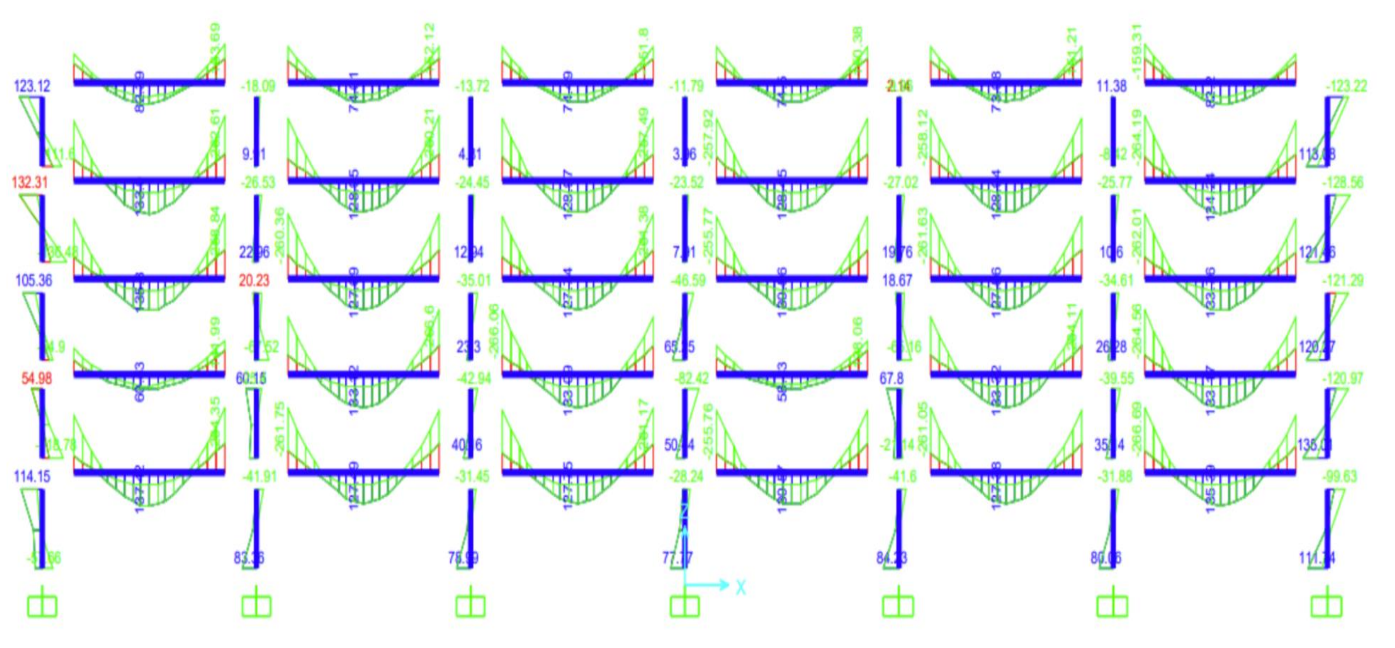
= 11.768"

b = 14"   (category A

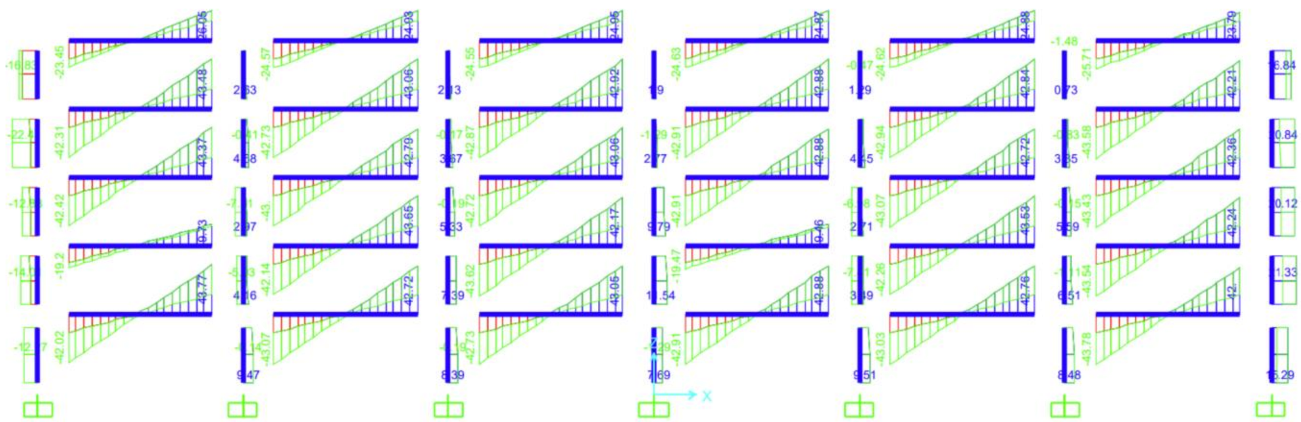


Results:

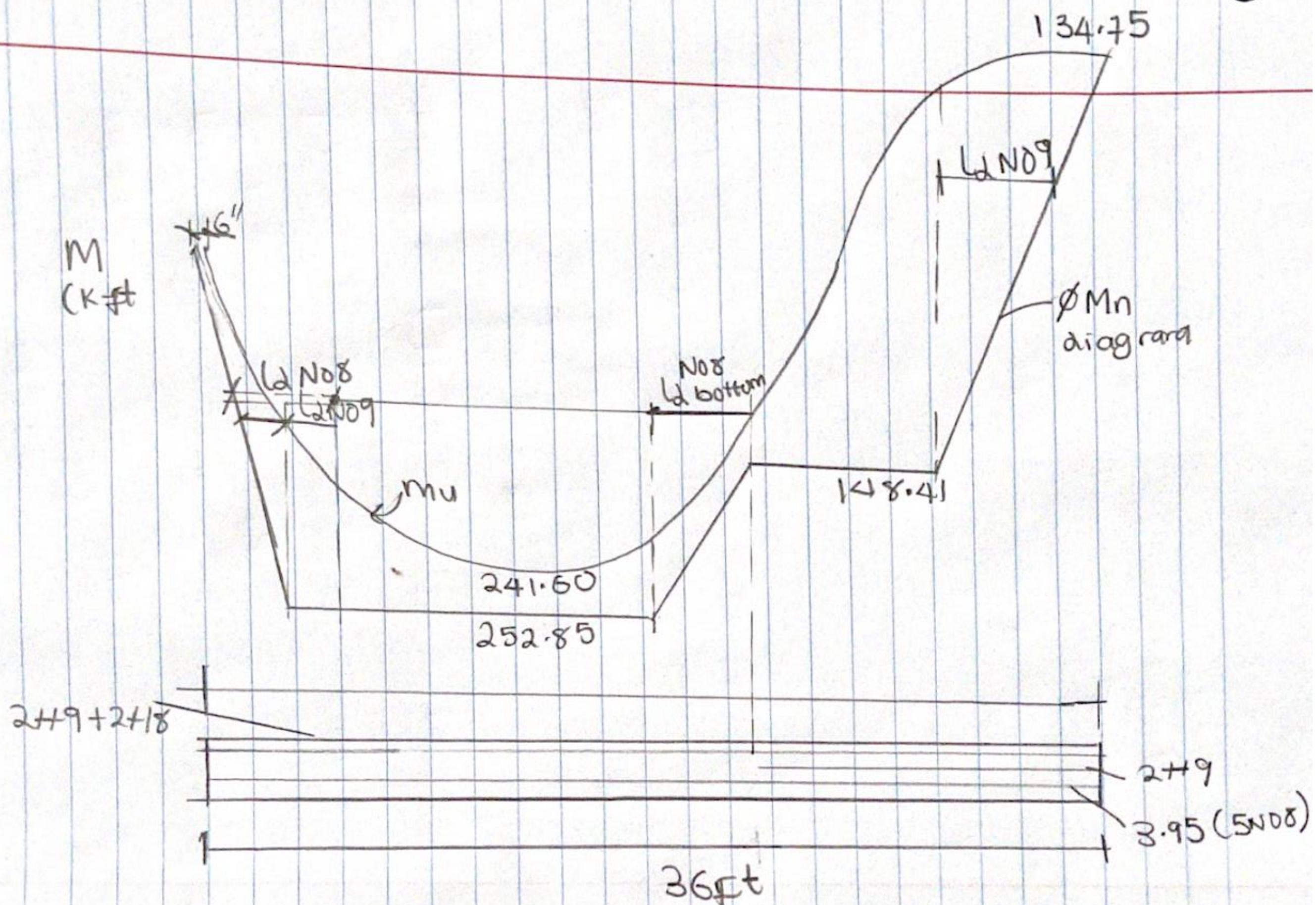
Moment Diagram



Shear Diagram

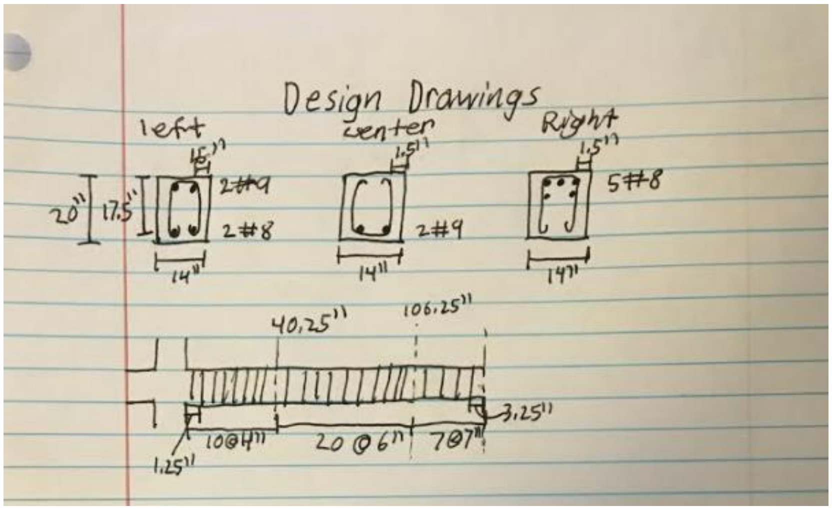


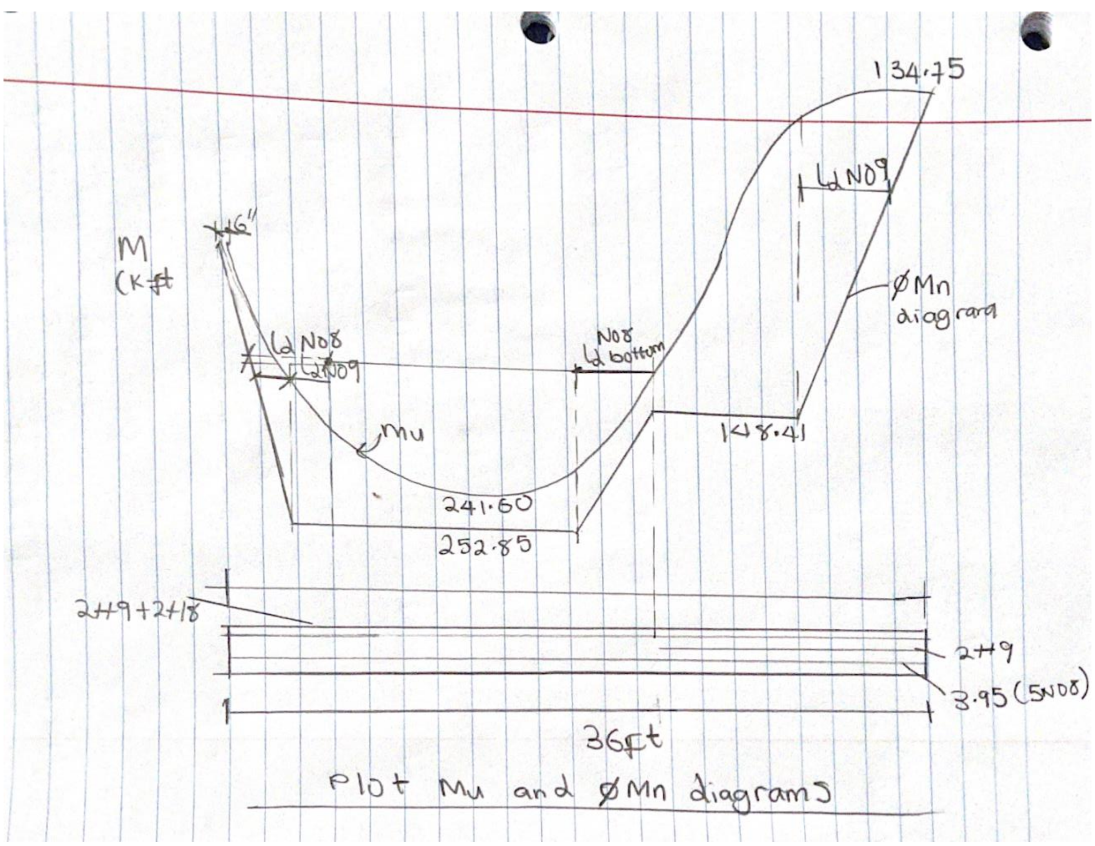
Longitudinal Bar With Moments



Plot Mu and ØMn diagrams

Design Drawings





Relevant Values:

Beam Cross Section	Height	Depth	Width
	20”	17.5”	14”
	Left	Center	Right
M(Kips-ft)	241.6	134.75	269.27
NO.OF BARS	2#9, 2#8	2#9	5#8
As(in^2)	3.58	2.0	3.95

	S_1	S_2	S_3
Stirrups	4”	6”	7”
Number of	10	20	7

Longitudinal Bars	Top	Bottom
#9	70”	54”
#8	62”	48”

Discussion of Errors:

____For the stirrups some problems were encountered when calculating the spacing. The spacing exceeded the maximum spacing allowed for every point selected (ACI 318-19 Table 9.7.6.2.2). It is assumed that the Vu calculated is too low to reduce the spacing. Therefore, the exact spacing for stirrups are not calculated due to having low value of Vu.