CE 3300 - Term Project

A Truss Investigation

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Executive Summary:

TAG Consulting was tasked with evaluating various roof truss designs for a small warehouse project in upstate New York, and making a finalized recommendation based upon specific requirements. The initial designs considered were Howe and Pratt trusses, both of which consisted of 6 equal width panels that spanned a total horizontal length of 42 feet and had a peak height of 18 feet from the bottom chord. All elements in these preliminary designs must have a uniform cross-sections of 4 inches. As mentioned in the investigation, Howe and Pratt trusses are frequently used by structural engineers due to their simple geometry and efficient force dissipating capabilities. Through SAP 2000, the two trusses were modeled and a Linear Static Analysis was performed and analyzed.

In addition to the two initial trusses,TAG Consulting was requested to propose a third alternative design based on the results from the first two designs. After completing an analysis on all three roof trusses, TAG Consulting recommends to the client to utilize the alternative proposal of the Double Fink truss design. As seen in Table 2, the Double Fink design with a peak height of 18 feet and a spanning distance of 42 feet best balanced cost and performance factors between all design options.

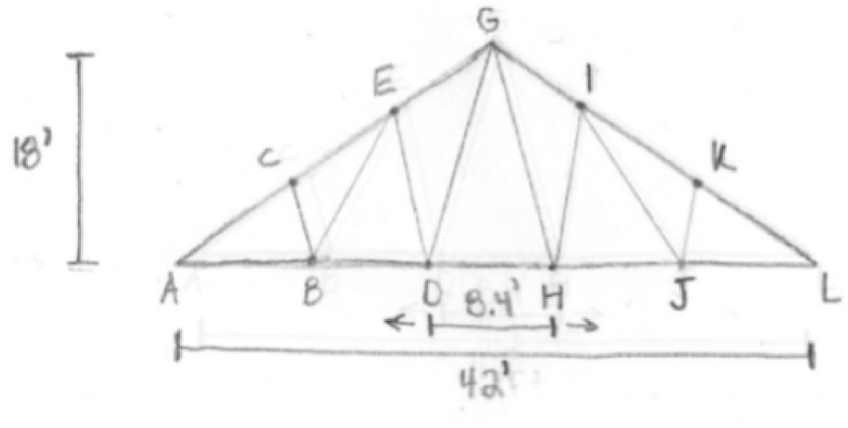


Figure 1. Final Double Fink Truss Design

Introduction:

As mentioned in the executive summary, the objective of this report was to analyze, evaluate, and select the best roof truss design for a small warehouse in upstate New York. The initial step in evaluating the best design is to calculate each load, dead, snow, ice, and roof live loads, imposed upon the truss. Calculation can be found in Appendix A, Figure 1. Research was conducted by another consulting firm that established the information necessary to calculate the roof dead and live loads, snow load, and the self-weight of the aluminum truss members. The Howe and Pratt trusses were modeled in SAP 2000 and a Linear Static Analysis was performed on each applicable factored load combinations, as specified in ASCE 7-10. Through the analysis, the maximum vertical deflection at each joint and the peak tensile and compressive member forces were evaluated for both trusses, as shown in Table 1. As per request, the cost and constructability were also considered, as seen in Table 3.

Following the rest of the report is a summary of how TAG Consulting approached the project and ultimately reached the final design. This starts by describing how each of the loads were calculated based upon prior research given. Any relevant results and figures will be provided through SAP 2000. The results of each respective design will be compared to each other to determine the best conclusion.

Further, the report will dive deeper with the TAG Consulting firm designing a third alternative. This process required research, collaboration and brainstorming amongst the entire team. Through supporting calculations and SAP 2000 figures, the firm can come to a final conclusion. The investigation will explain how TAG Consulting determined that a double fink truss with a height of 18 feet would most adequately support the roof of the warehouse in a cost effective and easily constructible way.

Problem Description:

For this project, our team was tasked with analyzing two potential roof truss designs for a small warehouse, as well as designing and analyzing a new truss design, to determine which design is best for load-bearing capacity as well as constructibility and cost factors. The two initial designs were Howe and Pratt trusses, two of the most common roof truss configurations. Each truss had a 42 foot span (divided into 6 equal panels of length 7 feet) and highest point 18 feet above the bottom chord, and were spaced 15 feet apart from each other. This configuration and the tributary area for each roof truss is displayed in Figure 2.

In determining which design was best able to bear the roof loadings, we looked at the maximum deflection values and the axial member forces for each truss design to determine the performance of

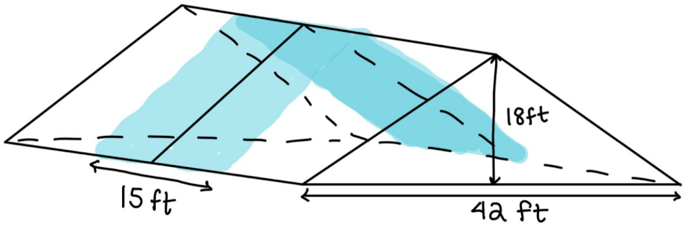


Figure 2. Basic truss configuration displaying the span, height, and spacing measurements as well as the roof tributary area for each truss shown in blue.

the design, while also considering the cost and ease of construction of each design. Our analytical process had two major components--first analyzing the given Howe and Pratt configurations and then analyzing other alternative designs. The first part of our analysis with the given trusses had a significant amount of design constraints, while the second part where we developed alternative designs only had constraints on the material (aluminum) and the span of the truss. Below, the methodology and steps taken to complete each portion of the analysis are detailed:

PART 1:

1. Hand calculations of specified loadings, including dead, snow, ice, and roof live load.
2. Hand calculations of material length and volume for the original Howe and Pratt truss designs
3. Computation of loading combinations according to ASCE-7
4. Analysis of Howe and Pratt trusses in SAP 2000 based on the maximum loading combination determined
5. Evaluation and comparison of the deflections and axial member forces of the Howe and Pratt trusses to the applied loading

PART 2:

6. Development of alternative truss designs based upon research and the results of the analysis in Part 1
7. Hand calculations of specified loadings for the alternative designs, particularly dead and live roof loads
8. Hand calculations of material length and volume for the alternative truss designs
9. Analysis of alternative designs in SAP 2000 based on the maximum loading combinations
10. Evaluation and comparison of the deflections, axial member forces, and costs for both the original and alternative truss designs to provide a comprehensive recommendation

Before beginning analyses of the two truss designs, it was necessary to look at the loadings they would be under, if implemented, to hold the roof up. Among these calculations include dead load, which was found by first determining the total length of aluminum included in the trusses, snow load, ice load and roof live load (Appendix A). At any given time, the roof of the warehouse could be under a combination of these loads, so we found the 7 most applicable load combinations according to ASCE 7 (Appendix A) and set out to find which of those combinations would exert the largest load on our trusses.

Below, the free body diagrams of both the initial Pratt and Howe trusses are displayed in Figures 3 and 4 after the application of the maximum loading combination from ASCE 7, determined in step 3 in Part 1 of our analysis.

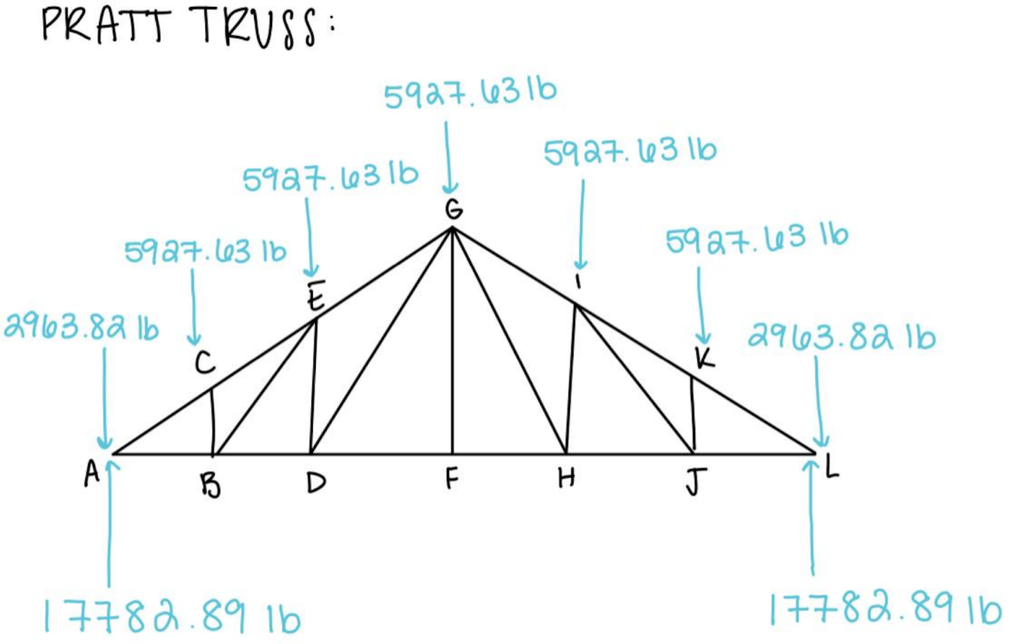


Figure 3. Free body diagram of the Pratt truss configuration with the maximum loading combination applied to it (ASCE 7 Combination 5: 1.2D + 1.6S)

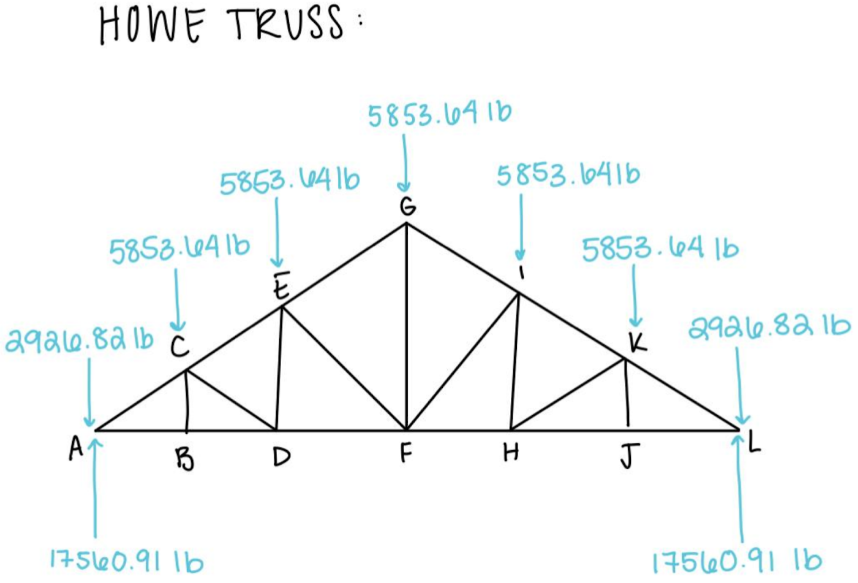


Figure 4. Free body diagram of the Howe truss configuration with the maximum loading combination applied to it (ASCE 7 Combination 5: 1.2D + 1.6S)

Since a significant portion of our project involved hand calculations of truss material quantities and loadings, our team needed to make some assumptions and apply some of the theory of analysis of planar trusses we have learned. All of the basic assumptions that come with planar truss analysis, such as assuming frictionless hinge connections at each truss joint and all loads are applied at truss joints, were made by our team during our analysis (Kassimali). We also assumed a square 4”x4” member cross section for all truss members and that a 6061-T6 aluminum alloy was used as the material, both in our loading and cost considerations.

Our team also had to consider basic structural analytical theory, particularly in how trusses transmit loads from their joints to their supports. As stated above, all loads and support reactions for a truss are applied at their joints. Because of this assumption, our team had to use structural analytical skills to determine how the distributed loadings we calculated were resolved into concentrated loads that could be transmitted through the trusses. We applied our understanding concepts like tributary areas and loading paths, as well as our understanding of how loads differ when applied to a horizontal area versus a sloped area, to do this.

Results:

The first portion of our analysis was analyzing and comparing the initial 18 foot Pratt and Howe trusses presented. Beginning with the standard Pratt truss, we found the maximum deflection of the truss to occur at Joints E and I and was a downward deflection of about 0.1128 inches, as shown in Figure 5a. The maximum internal axial forces occurred in 6 members of the Pratt truss, with peak tensile forces of 17289 lbs occuring in members AB and IL and peak compressive forces of 22771 lbs occuring in members AC, CE, IK, amd KL, as shown in Figure 5b.

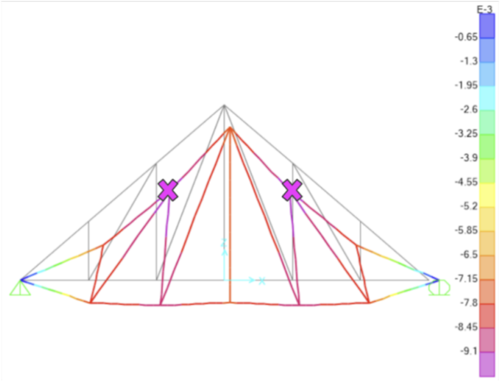


Figure 5a. Maximum deflection of the Pratt truss occurs at the points marked by the pink Xs and is a downward deflection of 0.1128 inches.

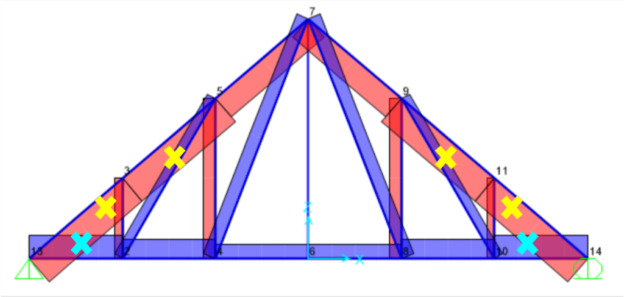


Figure 5b. Maximum tensile forces of 17289 lbs occur at the members marked by the blue Xs and maximum compressive forces of 22771 lbs occur in the members marked by the yellow Xs.

With the standard Howe truss, we found the maximum deflection of the truss to occur at Joint F and was a downward deflection of about 0.102 inches, as shown in Figure 6a. The maximum internal axial forces occurred in 6 members of the Howe truss, with peak tensile forces of 17073 lbs occuring in members AB, BD, HI and JL and peak compressive forces of 22487 lbs occuring in members AC and KL, as shown in Figure 6b.

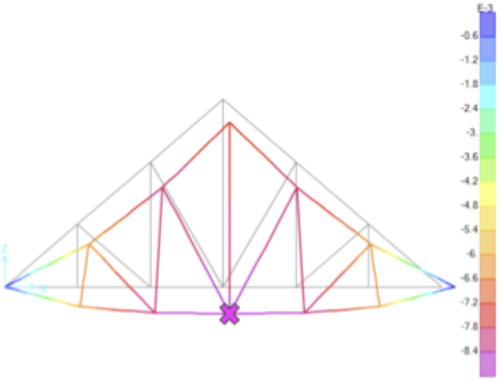


Figure 6a. Maximum deflection of the Howe truss occurs at the points marked by the pink Xs and is a downward deflection of 0.102 inches.

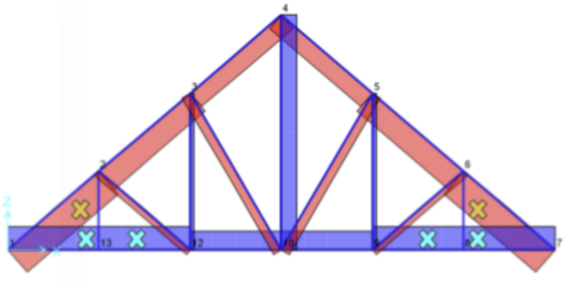


Figure 6b. Maximum tensile forces of 17073 lbs occur at the members marked by the blue Xs and maximum compressive forces of 22487 lbs occur in the members marked by the yellow Xs.

The completion of Part 1 of our analysis required an evaluation and comparison of the original Pratt and Howe truss designs. Comparing performance, the Howe truss very clearly outperformed the Pratt truss in both deflection and axial member forces. As seen in Table 1, the Howe truss had both lower maximum vertical deflection and lower maximum axial member forces (for both tension and compression). Cost-wise, the Howe truss was also the better option (for exactly how we completed the cost analysis for each design, see Appendix B). Thus, if we had to end our investigation here, the Howe truss would be our recommendation to the client. However, we wanted to investigate further to see if any variations of the Howe truss would outperform the initial design, as well as see if any variations of the Pratt truss would be better than the initial Howe truss.

Table 1: Standard Truss Performance Comparison

	Pratt Truss	Howe Truss
Maximum Deflection	0.1128 in.	0.102 in.
Max Deflection Joints	E, I	F
Peak Tensile Member Force (# of members)	17289 lbs (2)	17073 lbs (4)
Peak Compressive Member Force (# of members)	22771 lbs (4)	22487 lbs (2)

In developing a third, new design, we began with modifications of the original trusses–mainly, increasing or decreasing the height of the central member and changing the roof slope. In increasing the slope of the roof, we aimed to improve the performance of both the Howe and Pratt trusses by reducing the roof live load (which was dependent on the roof slope).

We also noticed that a lot of the internal forces transferring the load to the supports were carried in the perimeter members for both trusses; increasing the height of the truss also increases the length of the perimeter members, which we believed may distribute the load a bit better across the perimeter members. We increased the height of both the Pratt and Howe trusses to 24 feet, recalculated the dead loads (Appendix A, Figures 3a and 4b), and also analyzed them using the maximum loading combination of 1.2D +1.6S. The resulting maximum deflection and axial force values are displayed in Table 2. The qualitative SAP analyses figures show the joints at which these maximum deflections and peak axial forces are located (Appendix C, Figures 1a, 1b, 3a, and 3b)

We also considered what would happen if the slope of the roof was decreased. We hypothesized that doing so would drastically reduce the dead load being carried by the truss because a decrease in slope would lead to a decrease in peak height and thus a decrease in member length and volume. We believed that decreasing the total dead load within the truss would decrease the magnitude of the peak member forces as well as decrease the maximum deflection because the dead load is a large contributor to the maximum loading combination, 1.2D + 1.6S. The decrease in total member length would also lead to a large decrease in cost. Thus, we decided to test versions of the Pratt and Howe trusses where the peak height was set at 12 feet, recalculating the loading for each (Appendix A, Figures 3b and 4a). The resulting maximum deflection and peak member forces are displayed in Table 2. The qualitative SAP analyses figures show the joints at which these maximum deflections and peak axial forces are located (Appendix C, Figures 2a, 2b, 4a and 4b)

In addition, we did some research online to see if there were any other structurally sound truss designs fit for holding roofs which extend a horizontal distance of 42 feet. We ultimately went with an 18 foot tall Double Fink truss. Calculations for the loading on the truss can be found in Appendix A, Figure 5. Figures 7a and 7b show the maximum deflection and where it took place as well as the maximum tensile and compressive member forces and where they took place.

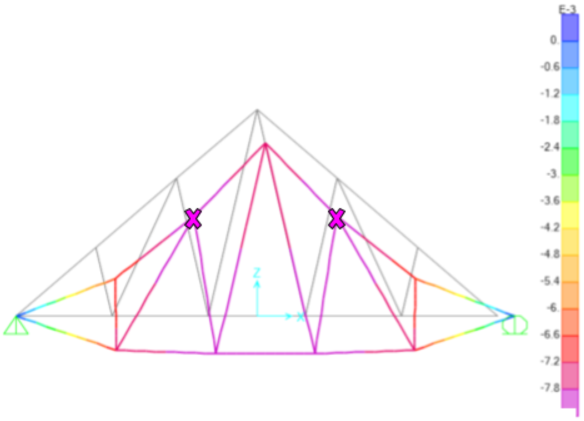


Figure 7a. Maximum deflection of the Double Fink truss occurs at the points marked by the pink Xs and is a downward deflection of 0.0996 inches.

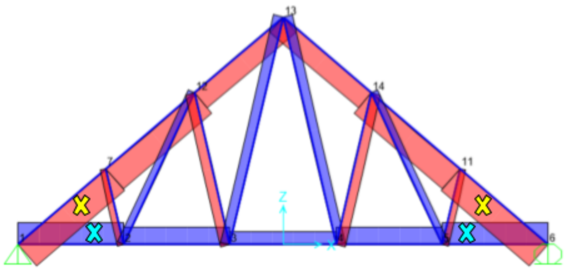


Figure 7b. Maximum tensile forces of 17075 lbs occur at the members marked by the blue Xs and maximum compressive forces of 22489 lbs occur in the members marked by the yellow Xs.

Although the Fink Truss would have been a better option as it has a much smaller total member length, it is not capable of providing adequate support in horizontal distances in excess of 30 feet (Minera). Besides its low total member length, and thus, ability to accommodate placement of storage within the truss given that the structure is a warehouse, we believed this truss design would be beneficial because it includes solely diagonal members which would transfer loads to the supports much better. It is worth noting that each member is bearing a compressive or tensile load while some members in previous designs don't carry any load. In addition, the decreased total member length would decrease the total cost of the truss. It also has one less member connection than the other designs which will decrease cost. After SAP analyses, the maximum deflection and peak tensile and compressive member forces were found and recorded below in Table 2.

Table 2: Alternate Truss Design Performance Comparison

	Howe (Tall)	Pratt (Tall)	Howe (Short)	Pratt (Short)	Double Fink
Max Deflection	0.0902 in.	0.1044 in.	0.1545 in.	0.1626 in.	0.0996 in.
Max Deflection Joins	F	E, I	F	E, I	E, I
Peak Tensile Member Force (# of members)	13408 lbs (4)	13638 lbs (2)	24564 lbs (4)	24744 lbs (2)	17075 lbs (2)
Peak Compressive Member Force (# of members)	20361 lbs (2)	20711 lbs (4)	28500 lbs (2)	28500 lbs (4)	22489 lbs (2)

Conclusions and Recommendations:

After taking a look at the performance of each truss design, a larger scale comparison was made with all designs and all factors applicable to the project were taken into account. Table 3, listed below, is the comparison of all the quantitative data that was collected and able to be compared between all of the trusses. It is important to note that we did not solely consult this table, we also took into account qualitative data about the trusses which factored into our final roof truss recommendation.

Table 3: Comparison of all Truss Designs

	Howe	Pratt	Howe (Tall)	Pratt (Tall)	Howe (Short)	Pratt (Short)	Double Fink
Total Cost	$36086	$39675	$42563	$47673	$30079	$32153	$36042
Max Deflection	0.102 in	0.1128 in.	0.0902 in.	0.1044 in.	0.1545 in.	0.1626 in.	0.0996 in.
Max Deflection Joins	F	E, I	F	E, I	F	E, I	E, I
Peak Tensile Member Force (# of members)	17073 lbs (2)	17289 lbs (2)	13408 lbs (4)	13638 lbs (2)	24564 lbs (4)	24744 lbs (2)	17075 lbs (2)
Peak Compressive Member Force (# of members)	22487 lbs (2)	22771 lbs (4)	20361 lbs (2)	20711 lbs (4)	28500 lbs (2)	28500 lbs (4)	22489 lbs (2)

The shorter Howe and Pratt trusses, while significantly less expensive than all the other options, also performed significantly worse in both maximum deflection and maximum member forces compared to all the other truss designs. Since the truss does need to hold up a roof, the less than optimal performance of these trusses cannot be ignored–especially since other designs performed much better. The taller variations of the Howe and Pratt trusses performed better than the originals and all the other designs–at significantly higher expense. Since the magnitude of the cost increase in these designs was not equally reflected in the increase in performance compared to the original Howe truss, we wanted to explore other options since a large increase in price for a moderate increase in performance is not an ideal solution, especially when scaling up these numbers for an entire warehouse.

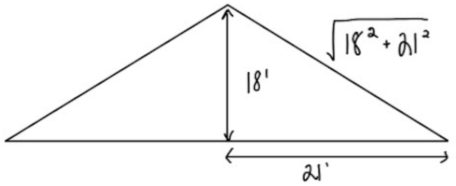
When left between the original Howe truss design and our Double Fink truss alternative, the Double Fink truss performed slightly better in both cost and performance. The Double Fink truss also had the loading better distributed across all the members in the configuration and had the majority of members in tension rather than compression. Especially for ductile materials, like the aluminum our truss is made of, tensile internal forces are preferable to compressive internal forces due to the two types of failure they induce. Compressive forces tend to result in buckling and more sudden modes of failure, while tensile forces tend to induce more gradual modes of failure. Gradual failure can be more easily detected and repaired compared to sudden failure modes.

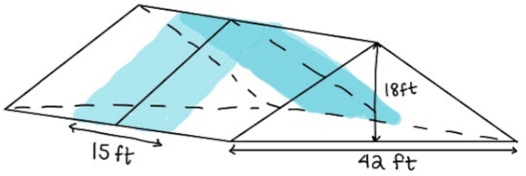
Thus, overall, we are recommending to the client the Double Fink truss design. This design best balanced cost and performance, as seen in the results in Table 2. It also distributed the loading across all of the members, meaning all the material used in the truss is working to transmit the load to the supports, and did so with the majority of forces in tension, creating a safer environment for failure detection.

Appendices:

Appendix A: Loading Calculations

1. loading calculations





tributary area calculations:

→ sloped area = $(\sqrt{18^2+21^2})(15)(2)$ = 829.76 ft^2

→ horizontal area = (21)(15)(2) = 630 ft^2

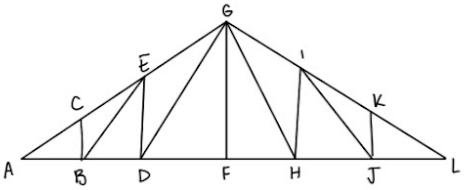
1A. Dead Load

Roofing & Decking

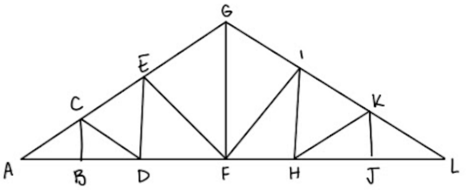
829.76 (5.6 lb/ft^2) = 4646.65 lb

truss members:

PRATT TRUSS:



HOWE TRUSS:



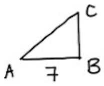
member ACEG = GIKL = $\sqrt{18^2+21^2}$ = 27.659 ft

member GF = 18 ft

members AB = BD = DF = FH = HJ = JL = 7 ft

members AC = CE = EG = GI = IK = KL = 9.22 ft

member CB & KJ



$\frac{AB}{CB} = \frac{AF}{GF}$

$CB = AB\left(\frac{GF}{AF}\right) = 7\left(\frac{18}{21}\right)$

KJ = CB = 6 ft

members ED & IH:

$\frac{AD}{ED} = \frac{AF}{GF} \longrightarrow ED = AD\left(\frac{GF}{AF}\right)$

$ED = 14\left(\frac{18}{21}\right)$

IH = ED = 12 ft

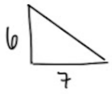
member CB & KJ:

KJ = CB = 6 ft

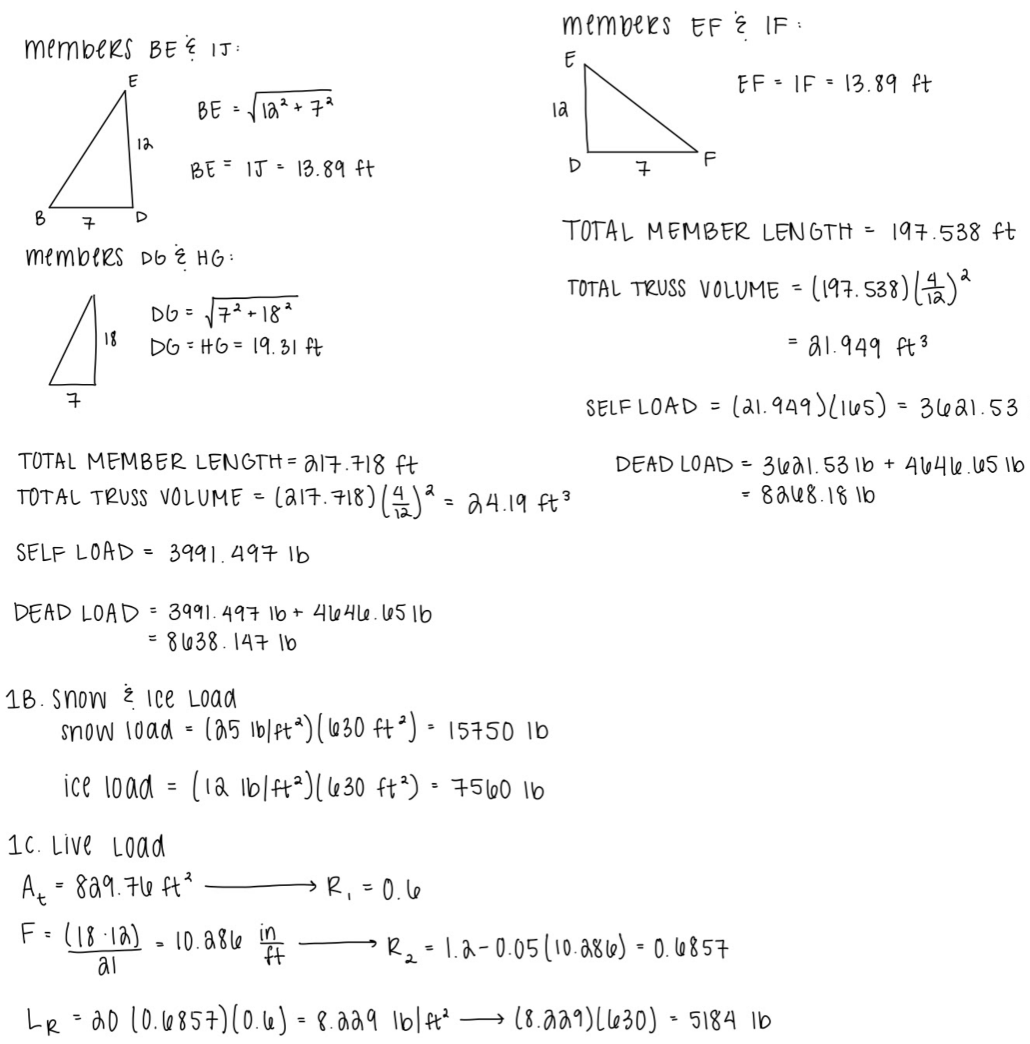
members ED & IH:

ED = IH = 12 ft

members CD & KH:



CD = KH = 9.22 ft



members BE & IJ:

E, B, D, 12, 7

$BE = \sqrt{12^2 + 7^2}$

$BE = IJ = 13.89$ ft

members DG & HG:

18, 7

$DG = \sqrt{7^2 + 18^2}$

$DG = HG = 19.31$ ft

TOTAL MEMBER LENGTH = 217.718 ft

TOTAL TRUSS VOLUME = $(217.718)\left(\frac{4}{12}\right)^2 = 24.19\ ft^3$

SELF LOAD = 3991.497 lb

DEAD LOAD = 3991.497 lb + 4646.65 lb
= 8638.147 lb

members EF & IF:

E, D, F, 12, 7

EF = IF = 13.89 ft

TOTAL MEMBER LENGTH = 197.538 ft

TOTAL TRUSS VOLUME = $(197.538)\left(\frac{4}{12}\right)^2$

$= 21.949\ ft^3$

SELF LOAD = (21.949)(165) = 3621.53 lb

DEAD LOAD = 3621.53 lb + 4646.65 lb
= 8268.18 lb

1B. snow & ice load

snow load = $(25\ lb/ft^2)(630\ ft^2) = 15750$ lb

ice load = $(12\ lb/ft^2)(630\ ft^2) = 7560$ lb

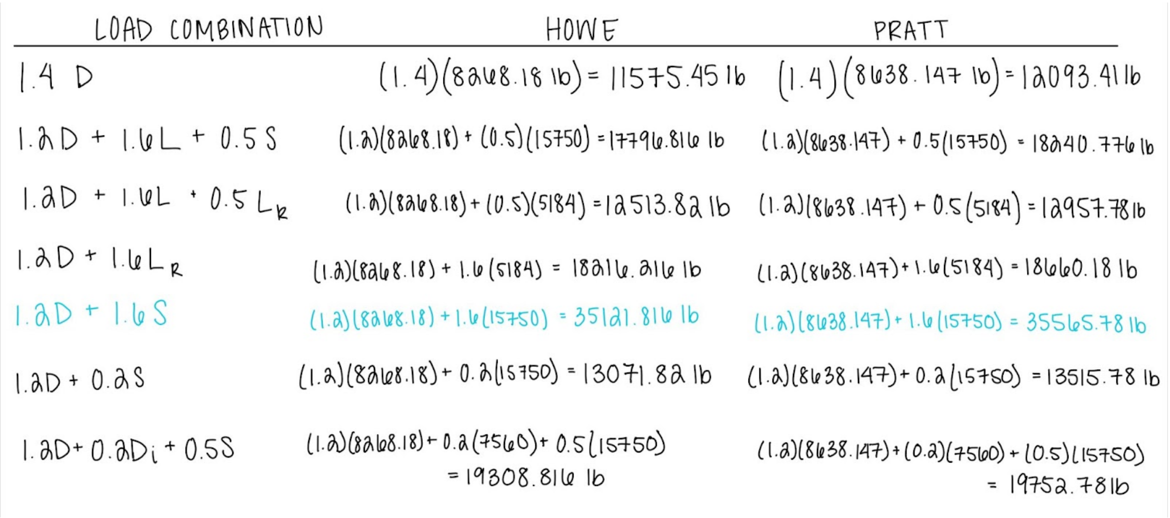
1C. Live Load

$A_t = 829.76\ ft^2 \longrightarrow R_1 = 0.6$

$F = \frac{(18 \cdot 12)}{21} = 10.286\ \frac{in}{ft} \longrightarrow R_2 = 1.2 - 0.05(10.286) = 0.6857$

$L_R = 20\ (0.6857)(0.6) = 8.229\ lb/ft^2 \longrightarrow (8.229)(630) = 5184$ lb

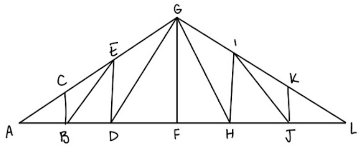
Figure 1. The basic loading calculations for the initial Howe and Pratt truss designs



LOAD COMBINATION	HOWE	PRATT
1.4 D	(1.4)(8268.18 lb) = 11575.45 lb	(1.4)(8638.147 lb) = 12093.41 lb
1.2D + 1.6L + 0.5 S	(1.2)(8268.18) + (0.5)(15750) = 17796.816 lb	(1.2)(8638.147) + 0.5(15750) = 18240.776 lb
1.2D + 1.6L + 0.5 L_R	(1.2)(8268.18) + (0.5)(5184) = 12513.82 lb	(1.2)(8638.147) + 0.5(5184) = 12957.78 lb
1.2D + 1.6L_R	(1.2)(8268.18) + 1.6(5184) = 18216.216 lb	(1.2)(8638.147) + 1.6(5184) = 18660.18 lb
1.2D + 1.6 S	(1.2)(8268.18) + 1.6(15750) = 35121.816 lb	(1.2)(8638.147) + 1.6(15750) = 35565.78 lb
1.2D + 0.2S	(1.2)(8268.18) + 0.2(15750) = 13071.82 lb	(1.2)(8638.147) + 0.2(15750) = 13515.78 lb
1.2D + 0.2D_i + 0.5S	(1.2)(8268.18) + 0.2(7560) + 0.5(15750) = 19308.816 lb	(1.2)(8638.147) + (0.2)(7560) + (0.5)(15750) = 19752.78 lb

Figure 2. Loading combinations for the initial truss designs based on ASCE 7. Note that the combination with the maximum effect is 1.2D + 1.6S, written in blue in the figure.

1. higher sloped Pratt truss



member GIKL = ACEG = $\sqrt{24^2 + 21^2}$ = 31.89 ft

member AC = CE = EG = GI = IK = KL = 10.63 ft

member AB = BD = DF = FH = HJ = JL = 7 ft

member CB = JK = 8 ft

member ED = IH = 16 ft

member GF = 24 ft

member BE = JI = $\sqrt{16^2 + 7^2}$ = 17.46 ft

member DG = HG = $\sqrt{24^2 + 7^2}$ = 25 ft

SLOPED TRIBUTARY:

A = (31.89)(15)(2) = 956.713 ft

TOTAL MEMBER LENGTH:

L = 42 + 24 + 2(31.89) + 2(8) + 2(16) + 2(17.46) + 2(25)

L = 262.7 ft

LOADING:

self load: $(262.7\ ft)\left(\frac{4}{12}\right)^2\left(165\ \frac{lb}{ft^3}\right) = 4816.167\ lb$

dead load of roof: $(956.713\ ft^2)\left(5.6\ \frac{lb}{ft^2}\right) = 5357.59\ lb$

total dead load: 4816.167 + 5357.59 = 10173.76 lb

snow load = $(25\ lb/ft^2)(630\ ft^2) = 15750\ lb$

ice load = $(12\ lb/ft^2)(630\ ft^2) = 7560\ lb$

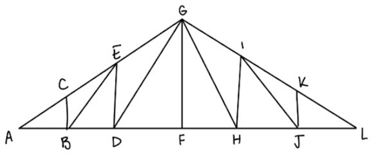
live roof load:

$$\left.\begin{array}{l} A_t = 956.713\ ft^2 \\ F = \frac{24 \cdot 12}{21} = 13.71 \end{array}\right] R_1 = R_2 = 0.6$$

$L_R = 20(0.6)(0.6) = 7.2\ lb/ft^2 \longrightarrow (7.2)(630) = 4536\ lb$

Figure 3a. Loading recalculations for the higher sloped, 24 foot Pratt truss. The dead load and roof live loads were the only loadings affected by this alternative design.

2. lower sloped Pratt truss



member GIKL = ACEG = $\sqrt{12^2 + 21^2}$ = 24.19 ft

member AC = CE = EG = GI = IK = KL = 8.06 ft

member AB = BD = DF = FH = HJ = JL = 7 ft

member CB = JK = 4 ft

member ED = IH = 8 ft

member GF = 12 ft

member BE = JI = $\sqrt{8^2 + 7^2}$ = 10.63 ft

member DG = HG = $\sqrt{12^2 + 7^2}$ = 13.89 ft

SLOPED TRIBUTARY:

A = (24.19)(15)(2) = 725.7 ft²

TOTAL MEMBER LENGTH:

L = 42 + 12 + 2(24.19) + 2(4) + 2(8) + 2(10.63) + 2(13.89)

L = 175.42 ft

LOADING:

self load: $(175.42\ ft)\left(\frac{4}{12}\right)^2\left(165\ \frac{lb}{ft^3}\right) = 3216.03\ lb$

dead load of roof: $(725.7\ ft^2)\left(5.6\ \frac{lb}{ft^2}\right) = 4063.92\ lb$

total dead load: 3216.03 + 4063.92 = 7279.9533 lb

snow load = $(25\ lb/ft^2)(630\ ft^2) = 15750\ lb$

ice load = $(12\ lb/ft^2)(630\ ft^2) = 7560\ lb$

live roof load:

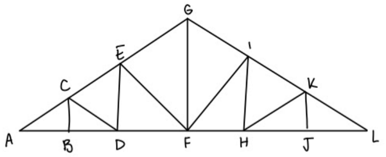
$A_t = 725.7\ ft^2 \longrightarrow R_1 = 0.6$

$F = \frac{12 \cdot 12}{21} = 6.857 \longrightarrow R_2 = 1.2 - 0.05(6.857) = 0.857$

$L_R = 20(0.6)(0.857) = 10.29\ lb/ft^2 \longrightarrow (10.29)(630) = 6480\ lb$

Figure 3b. Loading recalculations for the lower sloped, 12 foot Pratt truss. The dead load and roof live loads were the only loadings affected by this alternative design.

3. lower sloped Howe truss



member GIKL = ACEG = $\sqrt{12^2+21^2}$ = 24.19 ft

member AC = CE = EG = GI = IK = KL = 8.06 ft

member AB = BD = DF = FH = HJ = JL = 7 ft

member CB = JK = 4 ft

member ED = IH = 8 ft

member GF = 12 ft

member EF = FI = $\sqrt{8^2+7^2}$ = 10.63 ft

member CD = HK = $\sqrt{4^2+7^2}$ = 8.062 ft

SLOPED TRIBUTARY:

A = (24.19)(15)(2) = 725.7 ft^2

TOTAL MEMBER LENGTH:

L = 42 + 12 + 2(24.19) + 2(4) + 2(8)
+ 2(10.63) + 2(8.062)

L = 163.76 ft

LOADING:

self load: $(163.76\text{ ft})\left(\frac{4}{12}\right)^2\left(165\ \frac{\text{lb}}{\text{ft}^3}\right)$ = 3002.34 lb

dead load of roof: $(725.7\text{ ft}^2)\left(5.6\ \frac{\text{lb}}{\text{ft}^2}\right)$ = 4063.92 lb

total dead load: 3002.34 + 4063.92 = 7066.26 lb

snow load = (25 lb/ft^2)(630 ft^2) = 15750 lb

ice load = (12 lb/ft^2)(630 ft^2) = 7560 lb

live roof load:

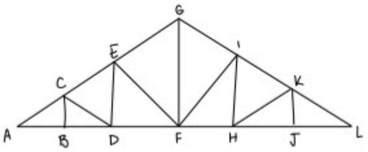
A_t = 725.7 ft^2 $\longrightarrow$ R_1 = 0.6

$F = \frac{12 \cdot 12}{21}$ = 6.857 $\longrightarrow$ R_2 = 1.2 − 0.05 (6.857) = 0.857

L_R = 20 (0.6)(0.857) = 10.29 lb/ft^2 $\longrightarrow$ (10.29)(630) = 6480 lb

Figure 4a. Loading recalculations for the lower sloped, 12 foot Howe truss. The dead load and roof live loads were the only loadings affected by this alternative design.

4. higher sloped Howe truss



member GIKL = ACEG = $\sqrt{24^2+21^2}$ = 31.89 ft

member AC = CE = EG = GI = IK = KL = 10.63 ft

member AB = BD = DF = FH = HJ = JL = 7 ft

member CB = JK = 8 ft

member ED = IH = 16 ft

member GF = 24 ft

member EF = FI = $\sqrt{16^2+7^2}$ = 17.46 ft

member CD = HK = $\sqrt{8^2+7^2}$ = 10.63 ft

SLOPED TRIBUTARY:

A = (31.89)(15)(2) = 956.713 ft

TOTAL MEMBER LENGTH:

L = 42 + 24 + 2(31.89) + 2(8) + 2(16)
+ 2(17.46) + 2(10.63)

L = 233.96 ft

LOADING:

self load: $(233.96\text{ ft})\left(\frac{4}{12}\right)^2\left(165\ \frac{\text{lb}}{\text{ft}^3}\right)$ = 4289.27 lb

dead load of roof: $(956.713\text{ ft}^2)\left(5.6\ \frac{\text{lb}}{\text{ft}^2}\right)$ = 5357.59 lb

total dead load: 4289.27 + 5357.59 = 9646.857 lb

snow load = (25 lb/ft^2)(630 ft^2) = 15750 lb

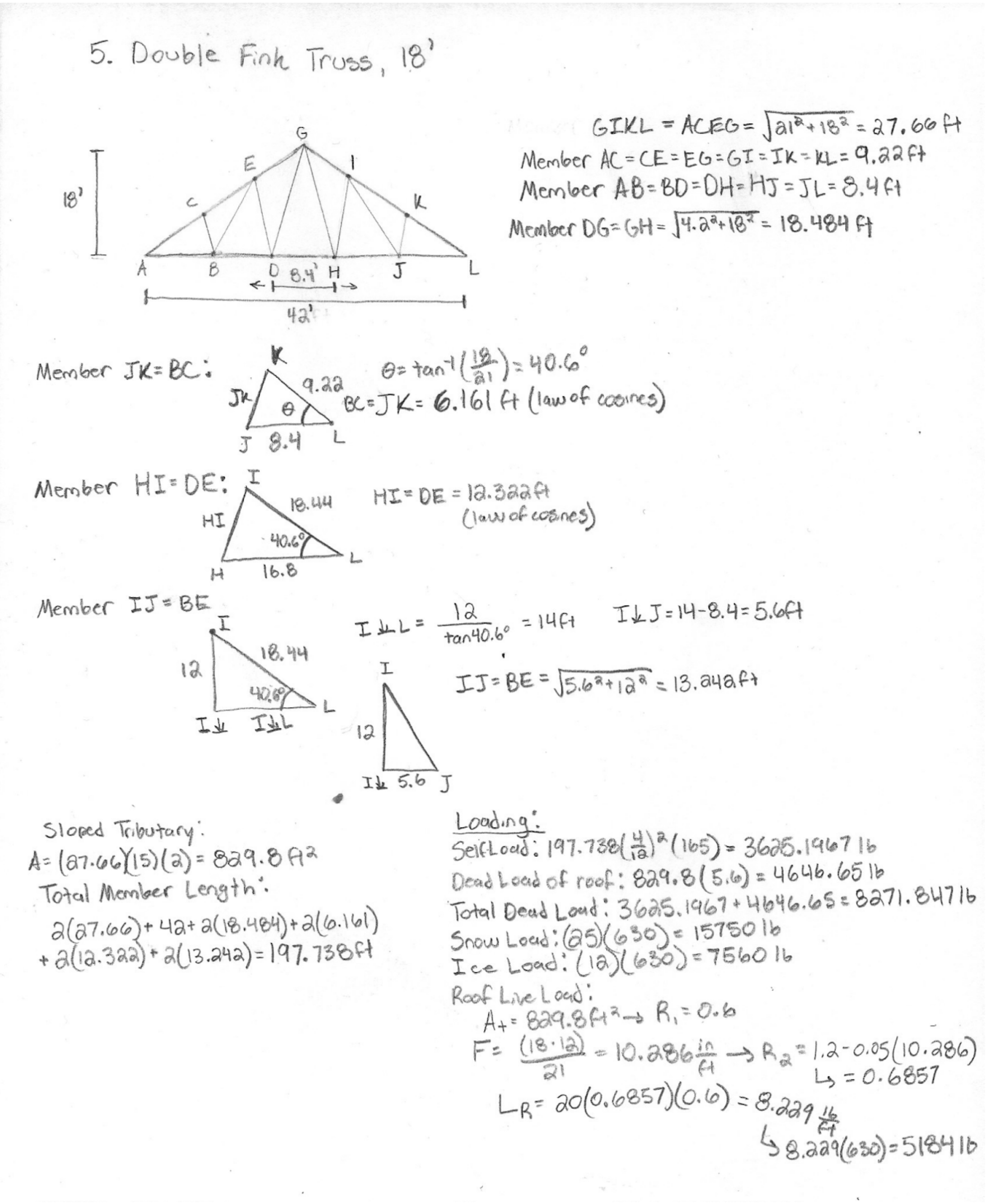
ice load = (12 lb/ft^2)(630 ft^2) = 7560 lb

live roof load:

$$\left.\begin{array}{l} A_t = 956.713\text{ ft}^2 \\ F = \frac{24 \cdot 12}{21} = 13.71 \end{array}\right] R_1 = R_2 = 0.6$$

L_R = 20 (0.6)(0.6) = 7.2 lb/ft^2 $\longrightarrow$ (7.2)(630) = 4536 lb

Figure 4b. Loading recalculations for the higher sloped, 24 foot Howe truss. The dead load and roof live loads were the only loadings affected by this alternative design.



5. Double Fink Truss, 18'

GIKL = ACEG = $\sqrt{21^2+18^2}$ = 27.66 ft

Member AC = CE = EG = GI = IK = KL = 9.22 ft

Member AB = BD = DH = HJ = JL = 8.4 ft

Member DG = GH = $\sqrt{4.2^2+18^2}$ = 18.484 ft

Member JK = BC:

$\theta = \tan^{-1}\left(\frac{18}{21}\right) = 40.6^\circ$

BC = JK = 6.161 ft (law of cosines)

Member HI = DE:

HI = DE = 12.322 ft (law of cosines)

Member IJ = BE

$I{\downarrow}L = \frac{12}{\tan 40.6^\circ} = 14$ ft

$I{\downarrow}J = 14 - 8.4 = 5.6$ ft

IJ = BE = $\sqrt{5.6^2+12^2}$ = 13.242 ft

Sloped Tributary:

$A = (27.66)(15)(2) = 829.8\ \text{ft}^2$

Total Member Length:

$2(27.66) + 42 + 2(18.484) + 2(6.161)$
$+ 2(12.322) + 2(13.242) = 197.738$ ft

Loading:

Self Load: $197.738\left(\frac{4}{12}\right)^2(165) = 3625.1967$ lb

Dead Load of roof: $829.8(5.6) = 4646.65$ lb

Total Dead Load: $3625.1967 + 4646.65 = 8271.847$ lb

Snow Load: $(25)(630) = 15750$ lb

Ice Load: $(12)(630) = 7560$ lb

Roof Live Load:

$A_t = 829.8\ \text{ft}^2 \rightarrow R_1 = 0.6$

$F = \frac{(18 \cdot 12)}{21} = 10.286\ \frac{\text{in}}{\text{ft}} \rightarrow R_2 = 1.2 - 0.05(10.286)$
$\hookrightarrow = 0.6857$

$L_R = 20(0.6857)(0.6) = 8.229\ \frac{\text{lb}}{\text{ft}}$
$\hookrightarrow 8.229(630) = 5184$ lb

Figure 5. Loading recalculations for the Double Fink truss. The dead load was the only loading affected by this alternative design.

Appendix B: Material and Cost Calculations

Cost of 4x4 in. aluminum member: $177.82/ft (Metals Depot)
Approximate Cost of Connection: $80 (Lee Valley)

	Pratt	Howe	Pratt (Tall)	Pratt (Short)	Howe (Tall)	Howe (Short)	Double Fink
Total Length (ft)	217.718	197.538	262.7	175.42	233.96	163.76	197.738
Total Cost of Aluminum	$38715	$35126	$46713	$31193	$41603	$29119	$35162
Member Connections	12	12	12	12	12	12	11
Total Cost of Connections	$960	$960	$960	$960	$960	$960	$880
Total Cost	$39675	$36086	$47673	$32153	$42563	$30079	$36042

Appendix C: Alternative Truss SAP 2000 Analysis Data

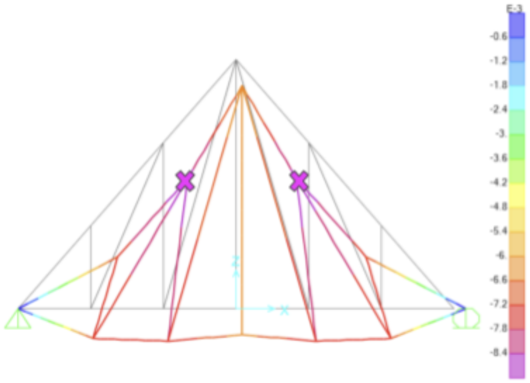


Figure 1a. Maximum deflection of the higher sloped, 24 foot Pratt truss occurs at the points marked by the pink Xs and is a downward deflection of 0.1044 inches.

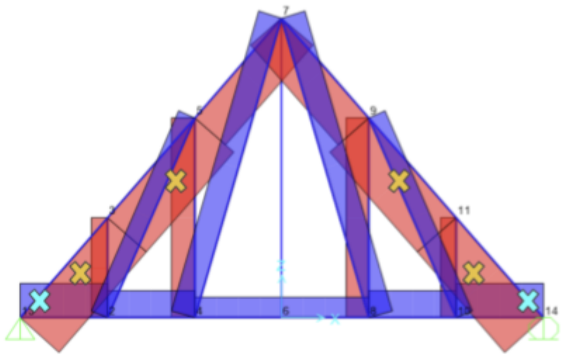


Figure 1b. Maximum tensile forces of 13638 lbs occur at the members marked by the blue Xs and maximum compressive forces of 20711 lbs occur in the members marked by the yellow Xs.

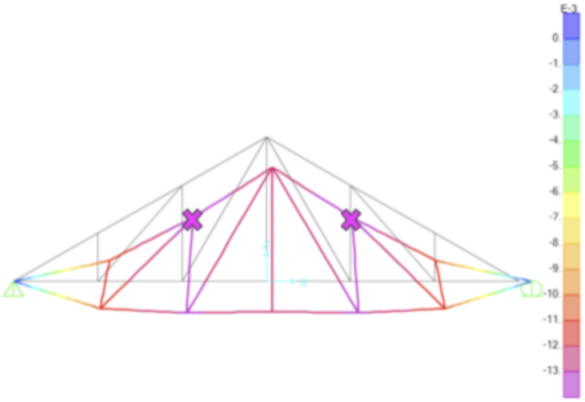


Figure 2a. Maximum deflection of the lower sloped, 12 foot Pratt truss occurs at the points marked by the pink Xs and is a downward deflection of 0.1626 inches.

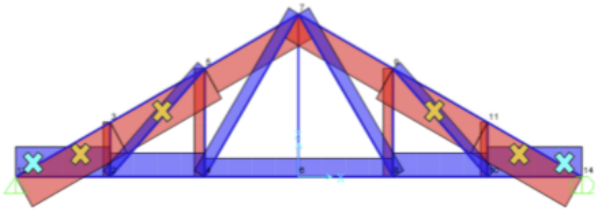


Figure 2b. Maximum tensile forces of 24744 lbs occur at the members marked by the blue Xs and maximum compressive forces of 28500 lbs occur in the members marked by the yellow Xs.

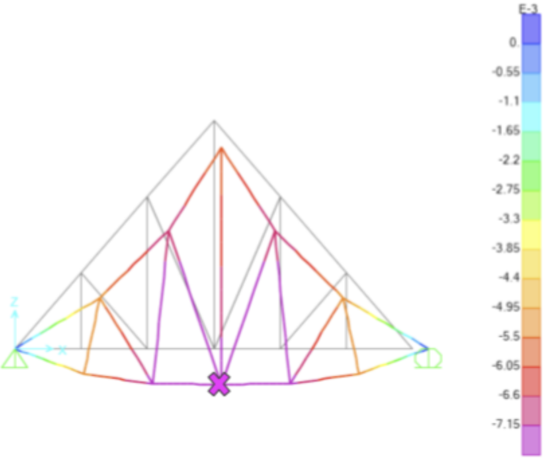


Figure 3a. Maximum deflection of the higher sloped, 24 foot Howe truss occurs at the points marked by the pink Xs and is a downward deflection of 0.0902 inches.

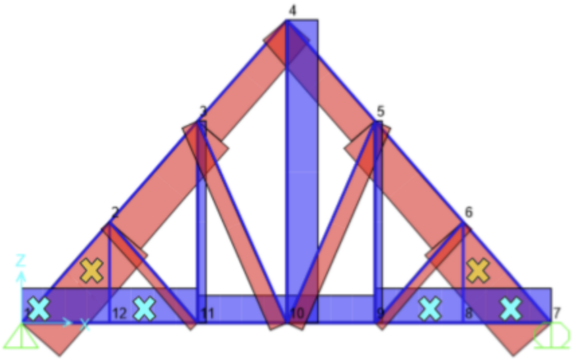


Figure 3b. Maximum tensile forces of 13408 lbs occur at the members marked by the blue Xs and maximum compressive forces of 20361 lbs occur in the members marked by the yellow Xs.

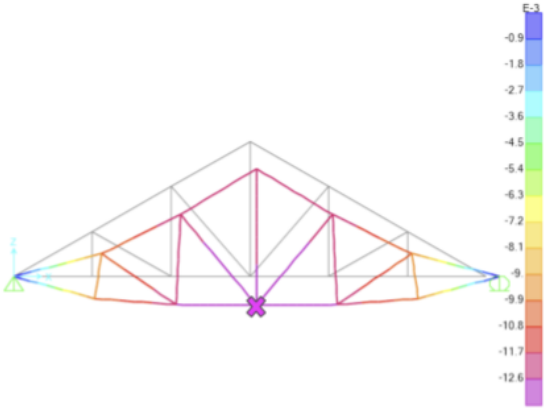


Figure 4a. Maximum deflection of the lower sloped, 12 foot Howe truss occurs at the points marked by the pink Xs and is a downward deflection of 0.1545 inches.

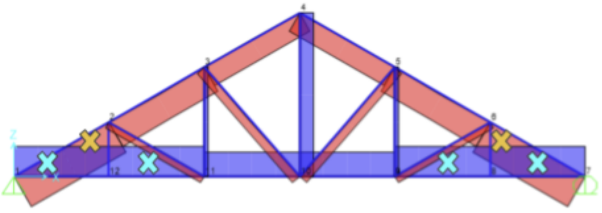


Figure 4b. Maximum tensile forces of 24564 lbs occur at the members marked by the blue Xs and maximum compressive forces of 28500 lbs occur in the members marked by the yellow Xs.

References:

Kassimali, A. (2020). Plane and Space Trusses. In Structural Analysis (6th ed., pp. 97–147). Cengage.

“MetalsDepot® - Aluminum Square 6061 - Shop Online!” *MetalsDepot® - Aluminum Square 6061 - Shop Online!*, https://www.metalsdepot.com/aluminum-products/aluminum-square-bar.

Minera Roof Trusses. “Types of Roof Trusses.” *Minera Roof Trusses*, https://www.minera-rooftrusses.com/types_of_roof_trusses.html.

“Ozco 8:12 Truss Base Fan.” *Google*, Google, https://www.google.com.vn/.