Project: Design of Experiments

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Experimental Design Considerations

Objective

For this experiment, our objective was to determine which factors affect the number of unpopped kernels when cooking microwave popcorn. To investigate the factors that significantly affect the response variable, three continuous factors were chosen. Factor one is the mass of the popcorn bag [X1] which had two levels: 50 grams and 90 grams. Factor two is the power of the microwave [X2] with two levels: 700 Watts and 1100 Watts. Factor three is the cooking time [X3] with three levels: 1.5 minutes, 2 minutes, and 2.5 minutes. The response [Y] is the number of unpopped kernels.

Two replicates were obtained for each factor level. We made sure to use the same devices for *both* level's replicates to ensure precision between runs. The tools used to quantify each of the factors were kept constant so that potential lurking variables were avoided. For the mass of popcorn, X1, the same brand, and the flavor of popcorn were used (PopSecret Original). For microwave power, X2, the brand of the microwave was kept constant (Toshiba). The time factor, X3, was controlled by inputting the desired cooking time separately for each run – instead of watching the countdown and manually stopping the timer which could have caused a delay. The response variable was measured by first carefully separating the already popped kernels, then proceeding to count the unpopped ones. Overall, the tools used maintained a high level of accuracy.

To perform a non-biased experiment, we used a Full Factorial design in JMP Pro Software that randomized the order of the runs. However, a potential lurking variable that was unable to be controlled is a possible variation between the locations where the original maize was grown. Because genetic modification practices in agriculture are common, some of the maize crops used by the company may be GMOs while others are not, which in turn could affect the individual kernel's likelihood to pop. Another potential lurking variable is the exact initial amount of unpopped kernels in the 50 g and 90 g bags. We tried our best to eliminate this by using only those two mass amounts exclusively, but it does not mean that all 50 g bags and 90 g bags of PopSecret have the same *number* of kernels, it is just a rough estimate of the total *mass* of kernels.

The experiment consisted of cooking the appropriate bag of popcorn at the specified power and time that each run required. After the corresponding time passed, the popcorn bag was removed from the microwave, and the number of unpopped kernels was counted and recorded. Since this was a 2x2x3 Factorial Design with n = 2 replicates, a total of twenty-four runs were done for this experiment. Doing this allows us to characterize variability.

Results and Discussion

Data Survey

The raw experimental results obtained are given in the following table and plot:

Factor Level	Popcorn bag mass (g)	Microwave Power (W)	Cooking time (minutes)	# of unpopped kernels	
[X1 X2 X3]	X1	X2	X3	Replicate 1	Replicate 2
---	50	700	1.5	66	72
+--	90	700	1.5	362	358
-+-	50	1100	1.5	27	30
++-	90	1100	1.5	151	148
--0	50	700	2	21	22
+-0	90	700	2	243	239
-+0	50	1100	2	1	2
++0	90	1100	2	34	39
+++	90	1100	2.5	8	10
--+	50	700	2.5	8	6
-++	50	1100	2.5	0	0
+-+	90	700	2.5	113	109

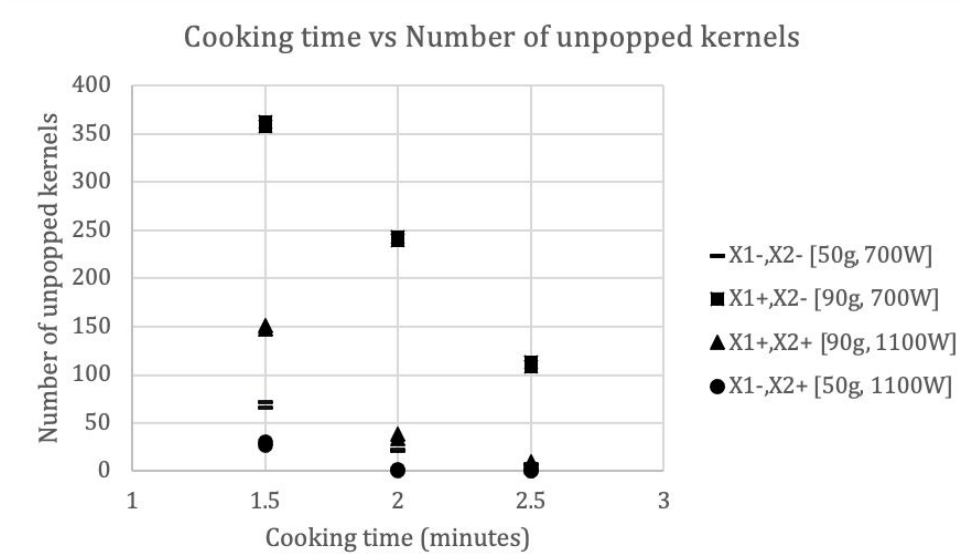


Figure 1 (Data Table and Experimental Plot)

After analyzing the graph shown in Figure 1, we can confidently report that cooking the popcorn for a longer period shows a decreasing trend in Response Y. When using the popcorn bag with the biggest mass [X1+] and the lowest microwave power [X2-], the response value increases on the y-axis (an indication of more unpopped kernels). When using the smallest popcorn bag [X1-] and the strongest microwave power [X2+], the response decreases (an indication of less unpopped kernels). The replicate that led to the most unpopped kernels occurred at the [+ - -] level, using the 90 g bag of popcorn, a 700 W microwave power, and cooking time of 1.5 minutes. Similarly, the replicate with the least amount of unpopped kernels occurred at the [- + +] level, using the 50 g bag of popcorn, an 1100 W microwave power, and a cooking time of 2.5 minutes.

Grand Average	86.20833333
Standard Pooled Dev.	2.389211864
Coefficient of Variation	0.027714396

Figure 2

As shown, we calculated the grand average, pooled standard deviation, and coefficient of variation in Figure 2. The coefficient of variation gave us a value of 0.0277, which means the variability within our response that *cannot* be explained is very low.

DOE Model

Our next step was to provide a model that would show the correlation between the factors that affected the response and how. To achieve this, we established the appropriate model for a 2x2x3 factorial design and used JMP pro to determine the parameters corresponding to our model.

Parameter Estimates					
"b"	**Estimate**	**Std Error**	**t Ratio**	**Prob > |t|**	**Significant? Yes/No**
b0	75.125	3.71	20.25	<.0001*	Yes
b1	64.958	2.142	30	<.0001*	Yes
b2	-48.708	2.142	-22.74	<.0001*	Yes
b3	-60	2.623	-22.87	<.0001*	Yes
b12	-37.458	2.142	-17.49	<.0001*	Yes
b23	17.75	2.623	6.77	<.0001*	Yes
b13	-37.375	2.623	-14.25	<.0001*	Yes
b123	9.375	2.623	3.57	0.0028	Yes
b33	16.625	4.543	3.66	0.0023	Yes

$$\hat{y} = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_{12}X_1X_2 + b_{23}X_2X_3 + b_{13}X_1X_3 + b_{123}X_1X_2X_3 + b_{33}X_3^2$$

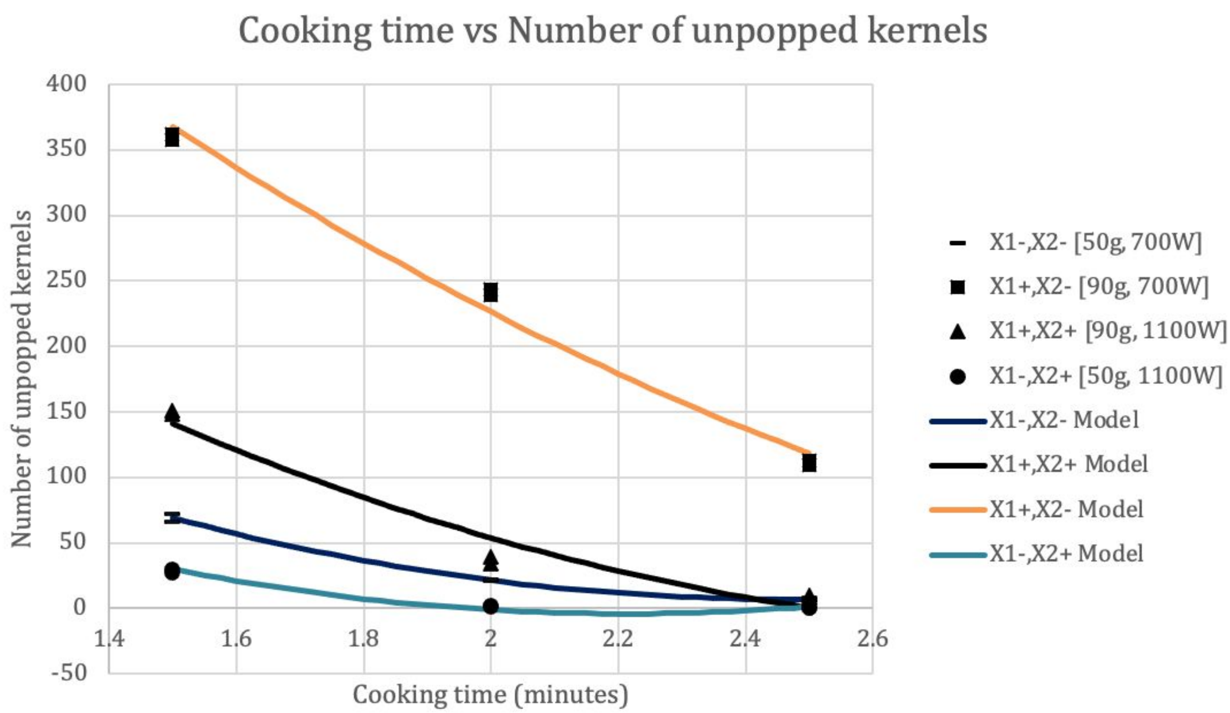
$$\hat{y} = 75.125 + 64.958X_1 - 48.708X_2 - 60X_3 - 37.458X_1X_2 - 37.375X_1X_3 + 17.75X_2X_3 + 9.375X_1X_2X_3 + 16.625X_3^2$$

R square	0.994132

Figure 3

Our model consisted of a 2x2x3 factorial design in which all of our main effects, interaction effects, and quadratic effects turned out to be statistically significant (with an alpha value less than 0.05). After obtaining our DOE model, we graphed the following plot to have a visual representation of the correlations.

Our R-squared value was calculated to be 0.99 using JMP Pro, which allows us to validate the model as one that is a good fit. Since we have a b_{33} term that is statistically significant, we should expect quadratic models. The model predicts that if we were to use the center of our range of y-values from the experimental data, we would have an average response of 75.125 unpopped kernels.



RMSE
8.294702

RMSE/grand average	0.096217

Figure 4

Figure 4 allows us to observe that all models are slightly concave up in shape. As the x-value increases, the measured response Y decreases. An interpretation is that no matter the factor combination, we will always see a decline in the number of unpopped kernels when the cooking time is increased. Furthermore, if we were to find the center point for the models and create an overall "average model," a projected y-intercept of 75.125 from the average model aligns with the visual data representation. The RMSE value was calculated to be 8.294 by squaring the sum of the residual, dividing by the number of replicates, then taking the square root. The R-squared value for the model is 0.99 which implies a good fit, but an additional way we can prove the validity of our model is by obtaining a value for RMSE/grand average. This comes out to be 0.096 shown in Figure 4. This is a small ratio and signifies that RMSE is minimal compared to our grand average, further strengthening the validity of our model.

Conclusions and Recommendations

After analyzing the model parameters illustrated in Figure 3, we can conclude the relative extent of response dependency on each factor. For example, out of the main effects, popcorn mass (factor X1) has the strongest effect on the measured response because b_1 is the term with the greatest magnitude within our parameter equation. This is a positive correlation so the bigger the popcorn mass, the more unpopped kernels. Cooking time (X3) has the second strongest effect on the response with a b_3 value of -60. As mentioned previously, more time cooking will lead to less unpopped kernels. Finally, microwave power (X2) has the smallest effect on the response. By also being a negative correlation, we found that the more powerful the microwave, the less unpopped kernels. The strongest interaction effect is represented by b_{12}. Although there is a varying degree to how much each factor influences the experimental outcome, all factors do significantly affect the response.

Every interaction effect, characterized in Figure 3 by b_{12}, b_{23}, b_{13}, and b_{123}, is significant. The interaction between power and mass (X1*X2) is the strongest because of its high magnitude compared to the other interaction terms. The interaction between mass and time (X1*X3) is the second strongest effect because it has the second to highest magnitude, represented by b_{13}. Interactions between power and time (X2*X3) had the second to lowest effect on the response, with the mass power and timer interaction (X1*X2*X3) having the least effect on the response.

The quadratic effect, represented by $b_{33}X_3$, also has a significant effect on the experimental outcome. This specific interaction term is what gives our model a concave shape.

From the conclusions, we can provide recommendations for those interested in recreating the design and obtaining certain results. To get the minimum number of unpopped kernels, which is likely the desired outcome for most individuals, one should use a factor combination that involves a 50 g bag of popcorn, a microwave with power 1100 W, and a cooking time of 2.5 minutes. The teal-colored model in Figure 4 (X1- X2+ Model) has the lowest range of y-values out of the four, which further supports this suggestion. To ensure precision between replicates, we suggest that the popcorn bag is placed in the center of the microwave each time so that the initial position remains constant. It is also recommended to allow the microwave to cool down for a predetermined amount of time between each run.